Chapter 9:

Static Equilibrium; Elasticity and Fracture

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9-1 The Conditions for Equilibrium

An object with forces acting on it, but that is not moving, is said to be in equilibrium.

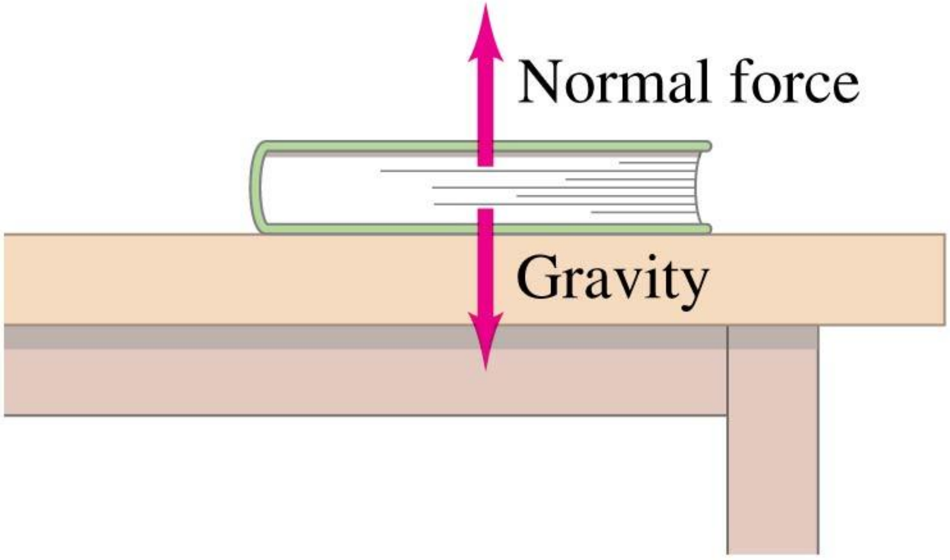
The First Condition for Equilibrium

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma F_z = 0.$$

$$\mathbf{\Sigma F_y = 0}$$

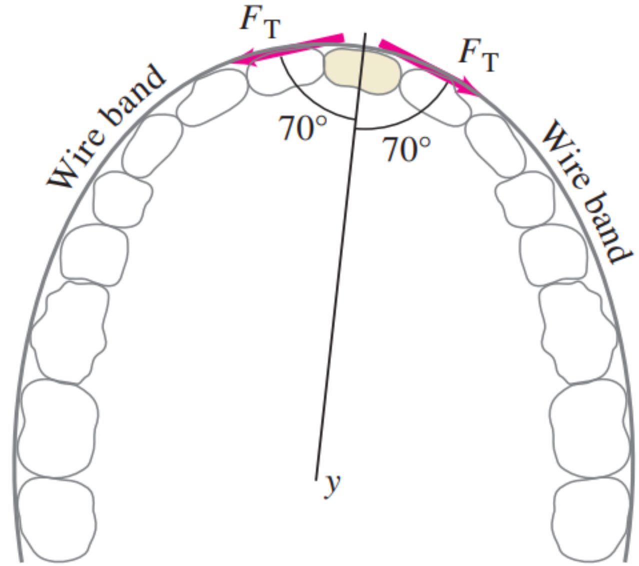
$$\mathbf{F_N - mg = 0}$$

$$\mathbf{F_N = mg}$$



EXAMPLE 9-1

Straightening teeth. The wire band shown in Fig. 9–3a has a tension of 2.0 N along it. It therefore exerts forces of 2.0 N on the highlighted tooth (to which it is attached) in the two directions shown. Calculate the resultant force on the tooth due to the wire, FR .



(a)

Solution:

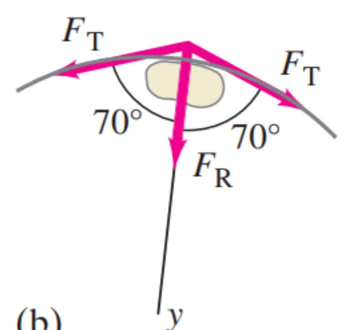
APPROACH Since the two forces F_T are equal, their sum will be directed along the line that bisects the angle between them, which we have chosen to be the y axis. The x components of the two forces add up to zero.

$$F_R = 2.0\text{ N},\ F_W = ?$$

$$F_{Rx} = F_T \sin(70^o) - F_T \sin(70^o) = 0$$

$$F_{Ry} = F_T \cos(70^o) + F_T \cos(70^o)$$

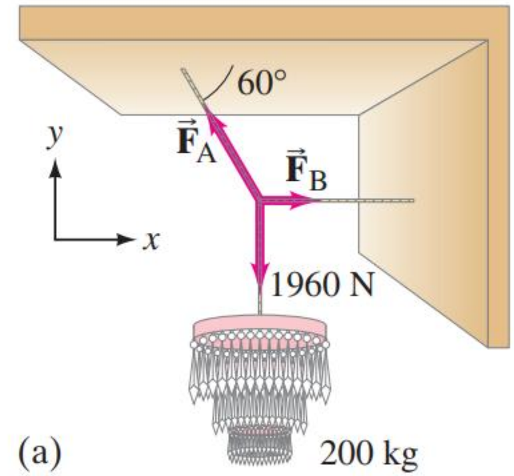
$$= 2F_T \cos(70^o) = 1.36 = 1.4\text{ N}$$



(b)

EXAMPLE 9-2

Chandelier cord tension. Calculate the tensions and in the two cords that are connected to the vertical cord supporting the 200-kg chandelier in Fig. 9–4. Ignore the mass of the cords



(a)

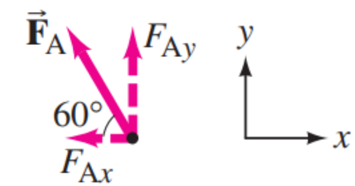
Solution:

$$mg = (200\,\text{kg})(9.80\,\text{m/s}^2) = 1960\,\text{N}.$$

$$\Sigma F_y = 0$$

$$F_A \sin 60^\circ - (200\,\text{kg})(g) = 0$$

$$F_A = \frac{(200\,\text{kg})g}{\sin 60^\circ} = (231\,\text{kg})g = (231\,\text{kg})(9.80\,\text{m/s}^2) = 2260\,\text{N}.$$



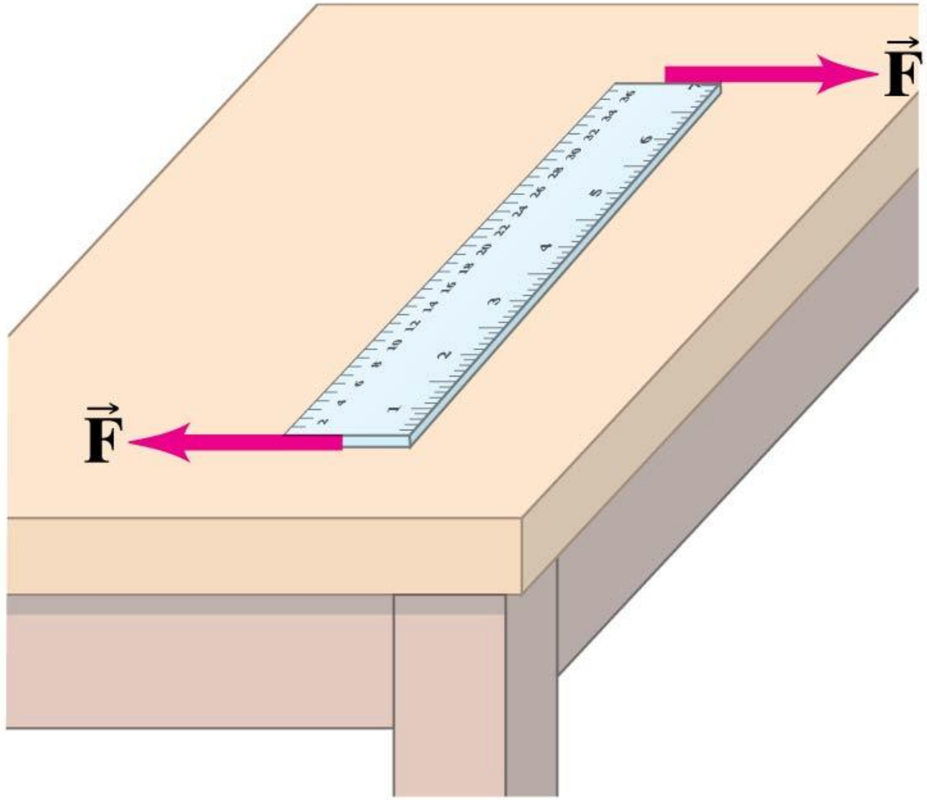
(b)

In the horizontal direction, with $\Sigma F_x = 0$,

$$\Sigma F_x = F_B - F_A \cos 60^\circ = 0.$$

Thus

$$F_B = F_A \cos 60^\circ = (231\,\text{kg})(g)(0.500) = (115\,\text{kg})g = 1130\,\text{N}.$$



Although the net force on it is zero, the ruler will move (rotate). A pair of equal forces acting in opposite directions but at different points on an object (as shown here) is referred to as a couple.

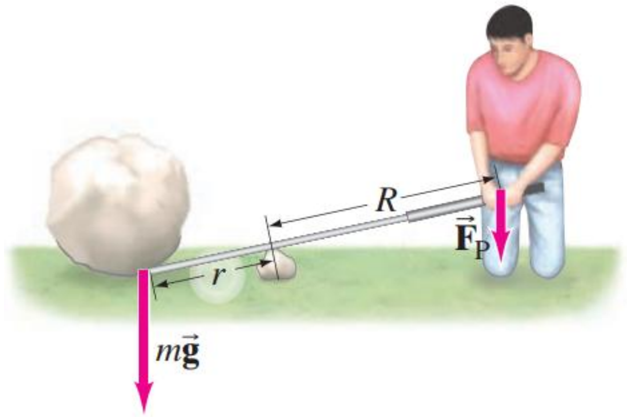
The Second Condition for Equilibrium

The second condition of equilibrium is that there be no torque around any axis; the choice of axis is arbitrary.

$$\Sigma\tau = 0.$$

EXAMPLE 9-3

A lever. The bar in Fig. 9–6 is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force required at the long end of the bar can be quite a bit smaller than the rock's weight mg, since it is the torques that balance in the rotation about the fulcrum. If, however, the leverage isn't sufficient, and the large rock isn't budged, what are two ways to increase the lever arm?



Solution:

In order to pry the rock, the torque due to F_P must at least balance the torque due to mg; that is

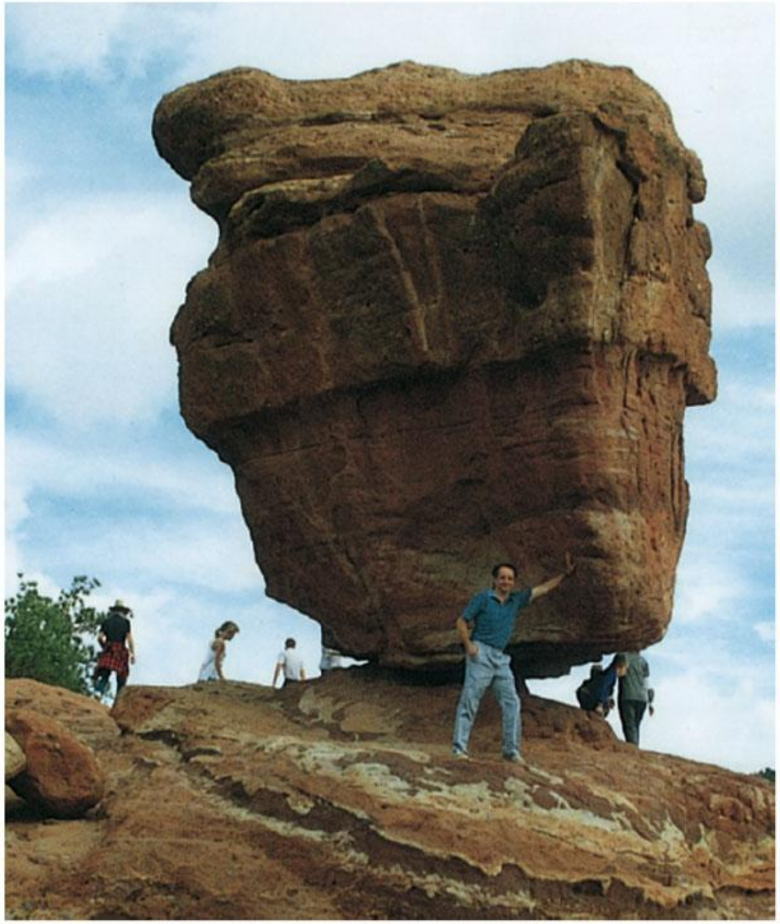
$$mgr = F_P R \text{ and}$$

$$\frac{r}{R} = \frac{F_P}{mg}.$$

With r smaller, the weight mg can be balanced with less force F_P.

Static Equilibrium

- Equilibrium and static equilibrium
- Static equilibrium conditions
  - Net external force must equal zero
  - Net external torque must equal zero
- Center of gravity
- Solving static equilibrium problems



Conditions for Equilibrium

- The net force equals zero

$$\sum \vec{\mathbf{F}} = 0$$

- The net torque equals zero

$$\sum \vec{\tau} = 0$$

- These conditions describe the rigid objects in the equilibrium analysis model

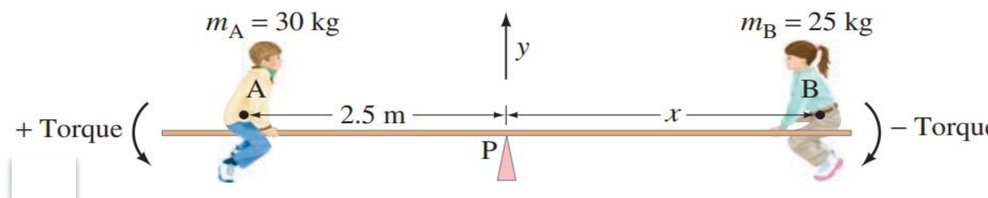
9-2 Solving Statics Problems

1. Choose one object at a time, and make a free-body diagram showing all the forces on it and where they act.
2. Choose a coordinate system and resolve forces into components.
3. Write equilibrium equations for the forces.
4. Choose any axis perpendicular to the plane of the forces and write the torque equilibrium equation. A clever choice here can simplify the problem enormously.
5. Solve.

EXAMPLE 9-4

Balancing a seesaw. A board of mass M = 4.0 kg serves as a seesaw for two children, as shown in Fig. 9–7a. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

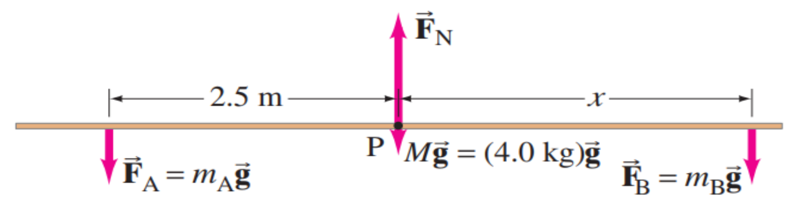
Solution:



$$\Sigma F_y = 0$$

$$F_N - m_A g - m_B g - Mg = 0,$$

where $F_A = m_A g$ and $F_B = m_B g$.



$$\Sigma \tau = 0$$

$$m_A g(2.5\text{ m}) - m_B g x + Mg(0\text{ m}) + F_N(0\text{ m}) = 0$$

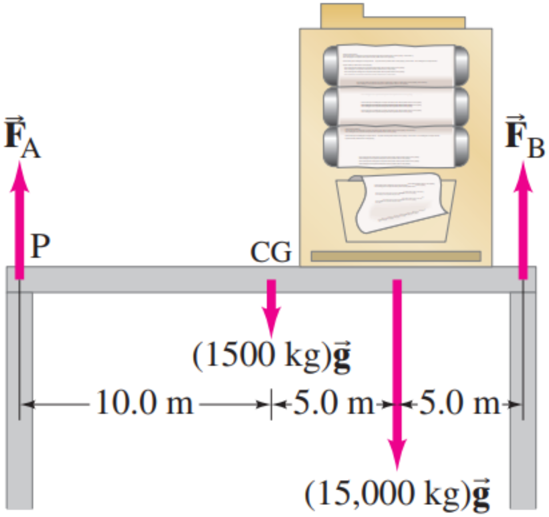
$$m_A g(2.5\text{ m}) - m_B g x = 0,$$

$$x = \frac{m_A}{m_B}(2.5\text{ m}) = \frac{30\text{ kg}}{25\text{ kg}}(2.5\text{ m}) = 3.0\text{ m}.$$

To balance the seesaw, child B must sit so that her CG is 3.0 m from the pivot point.

EXAMPLE 9-5

Forces on a beam and supports. A uniform 1500-kg beam, 20.0 m long, supports a 15,000-kg printing press 5.0 m from the right support column (Fig. 9–8). Calculate the force on each of the vertical support columns.



Solution:

We choose a convenient axis for writing the torque equation: the point of application of F_A (labeled P), so F_A will not enter the equation(its lever arm will be zero).

$$\Sigma\tau = 0, \qquad \Sigma\tau = -(10.0\,\text{m})(1500\,\text{kg})g - (15.0\,\text{m})(15{,}000\,\text{kg})g + (20.0\,\text{m})F_B = 0.$$

$$F_B = (12{,}000\,\text{kg})g = 118{,}000\,\text{N}.$$

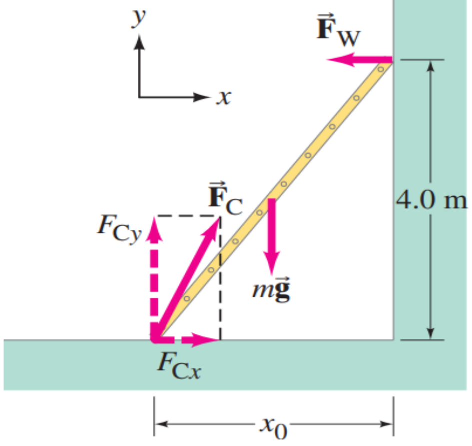
$$\Sigma F_y = 0, \text{ with } +y \text{ upward:}$$

$$\Sigma F_y = F_A - (1500\,\text{kg})g - (15{,}000\,\text{kg})g + F_B = 0.$$

$$F_A = (4500\,\text{kg})g = 44{,}100\,\text{N}.$$

EXAMPLE 9-7

Ladder. A 5.0-m-long ladder leans against a wall at a point 4.0 m above a cement floor as shown. The ladder is uniform and has mass m = 12.0 kg. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.



Solution:

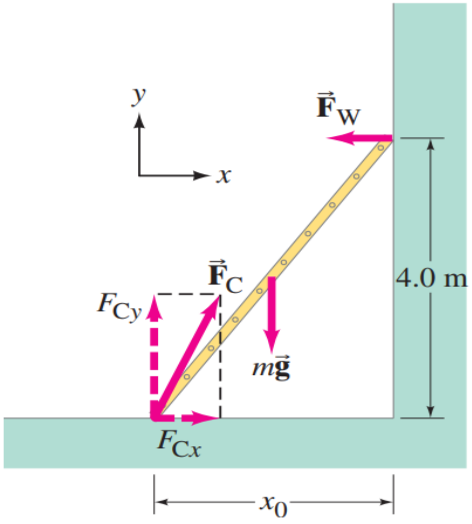
APPROACH Figure 9–11 is the free-body diagram for the ladder, showing all the forces acting on the ladder. The wall, since it is frictionless, can exert a force only perpendicular to the wall, and we label that force $\vec{\mathbf{F}}_W$. The cement floor exerts a force $\vec{\mathbf{F}}_C$ which has both horizontal and vertical force components: F_{Cx} is frictional and F_{Cy} is the normal force. Finally, gravity exerts a force $mg = (12.0\,\text{kg})(9.80\,\text{m/s}^2) = 118\,\text{N}$ on the ladder at its midpoint, since the ladder is uniform.

$$mg = (12.0\,\text{kg})(9.80\,\text{m/s}^2) = 118\,\text{N}$$

$$\Sigma F_y = F_{Cy} - mg = 0, \qquad F_{Cy} = mg = 118\,\text{N}.$$

EXAMPLE 9-7

$$x_0 = \sqrt{(5.0\,\text{m})^2 - (4.0\,\text{m})^2} = 3.0\,\text{m}$$



The lever arm for mg is half this, or 1.5 m, and the lever arm for F_W is 4.0 m

The torque equation about the ladder's contact point on the cement is

$$\Sigma\tau = (4.0\,\text{m})F_W - (1.5\,\text{m})mg = 0.$$

$$F_W = \frac{(1.5\,\text{m})(12.0\,\text{kg})(9.8\,\text{m/s}^2)}{4.0\,\text{m}} = 44\,\text{N}.$$

$$\Sigma F_x = F_{Cx} - F_W = 0.$$

$$F_{Cx} = F_W = 44\,\text{N}.$$

Since the components of $\vec{F}_C$ are $F_{Cx} = 44\,\text{N}$ and $F_{Cy} = 118\,\text{N}$, then

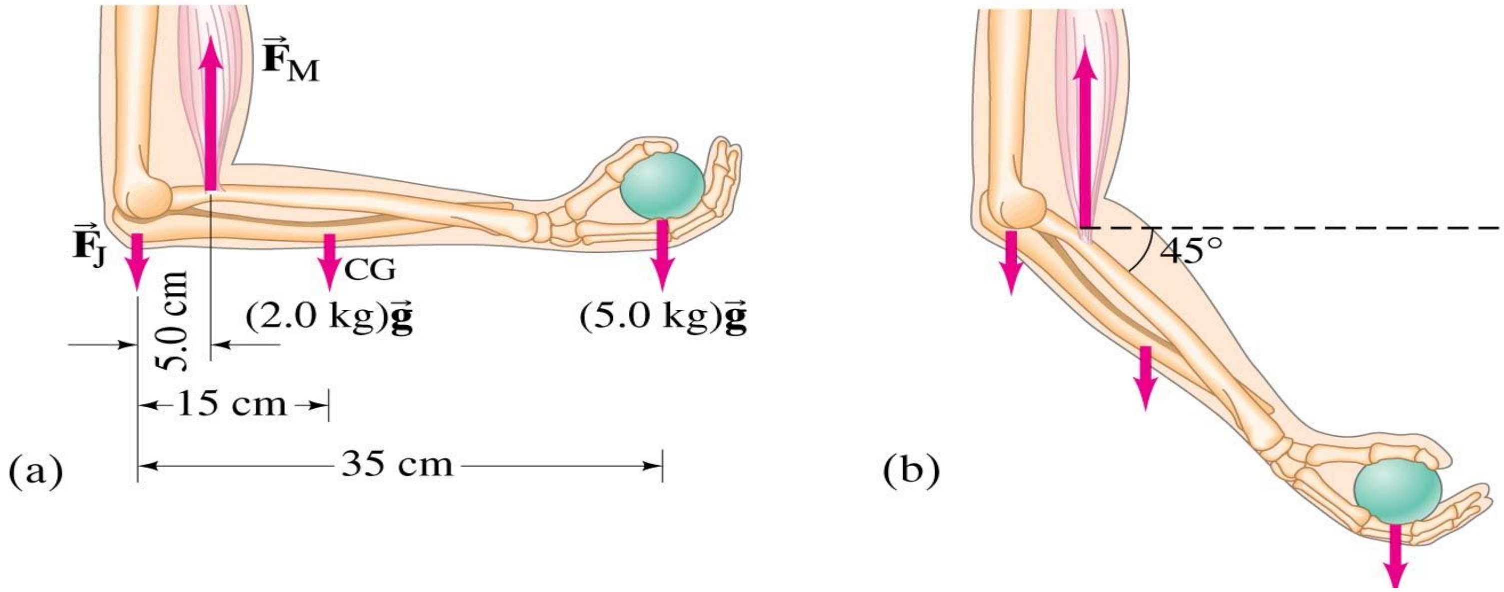
$$F_C = \sqrt{(44\,\text{N})^2 + (118\,\text{N})^2} = 126\,\text{N} \approx 130\,\text{N}$$

(rounded off to two significant figures), and it acts at an angle to the floor of

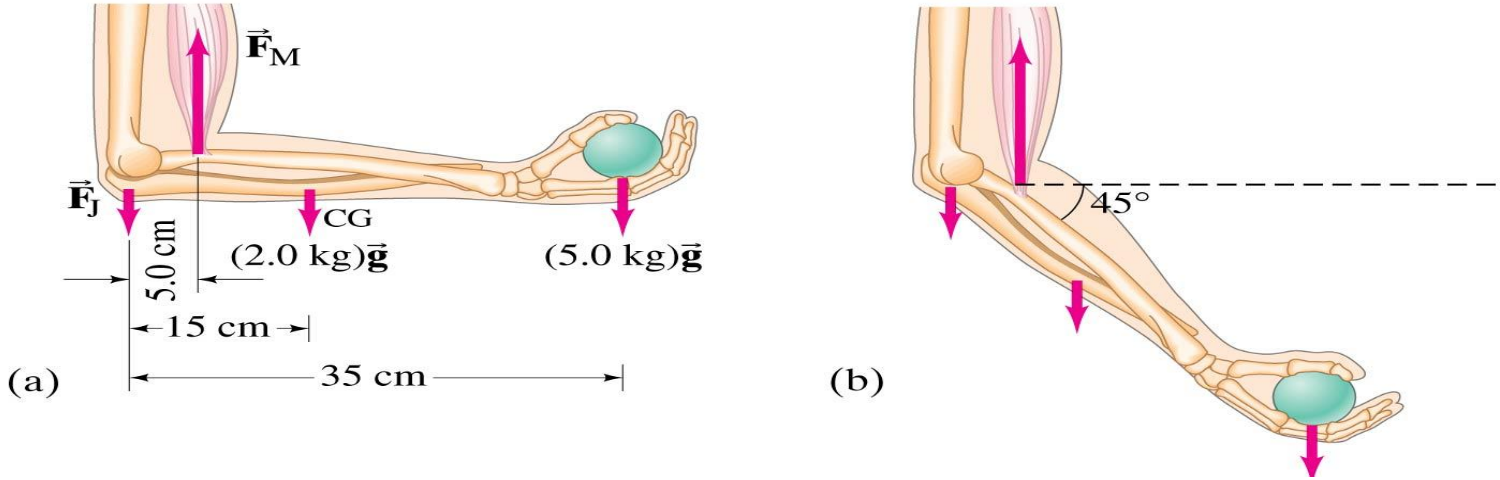
$$\theta = \tan^{-1}(118\,\text{N}/44\,\text{N}) = 70°.$$

EXAMPLE 9-8

Force exerted by biceps muscle. How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (a) with the arm horizontal as in Fig. 9–13a, and (b) when the arm is at a 45° angle as in Fig. 9–13b? The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0 kg and their CG is as shown.



EXAMPLE 9-8



SOLUTION (*a*) We calculate torques about the point where $\vec{\mathbf{F}}_J$ acts in Fig. 9–13a. The $\Sigma\tau = 0$ equation gives

$$(0.050\ \text{m})F_M - (0.15\ \text{m})(2.0\ \text{kg})g - (0.35\ \text{m})(5.0\ \text{kg})g = 0.$$

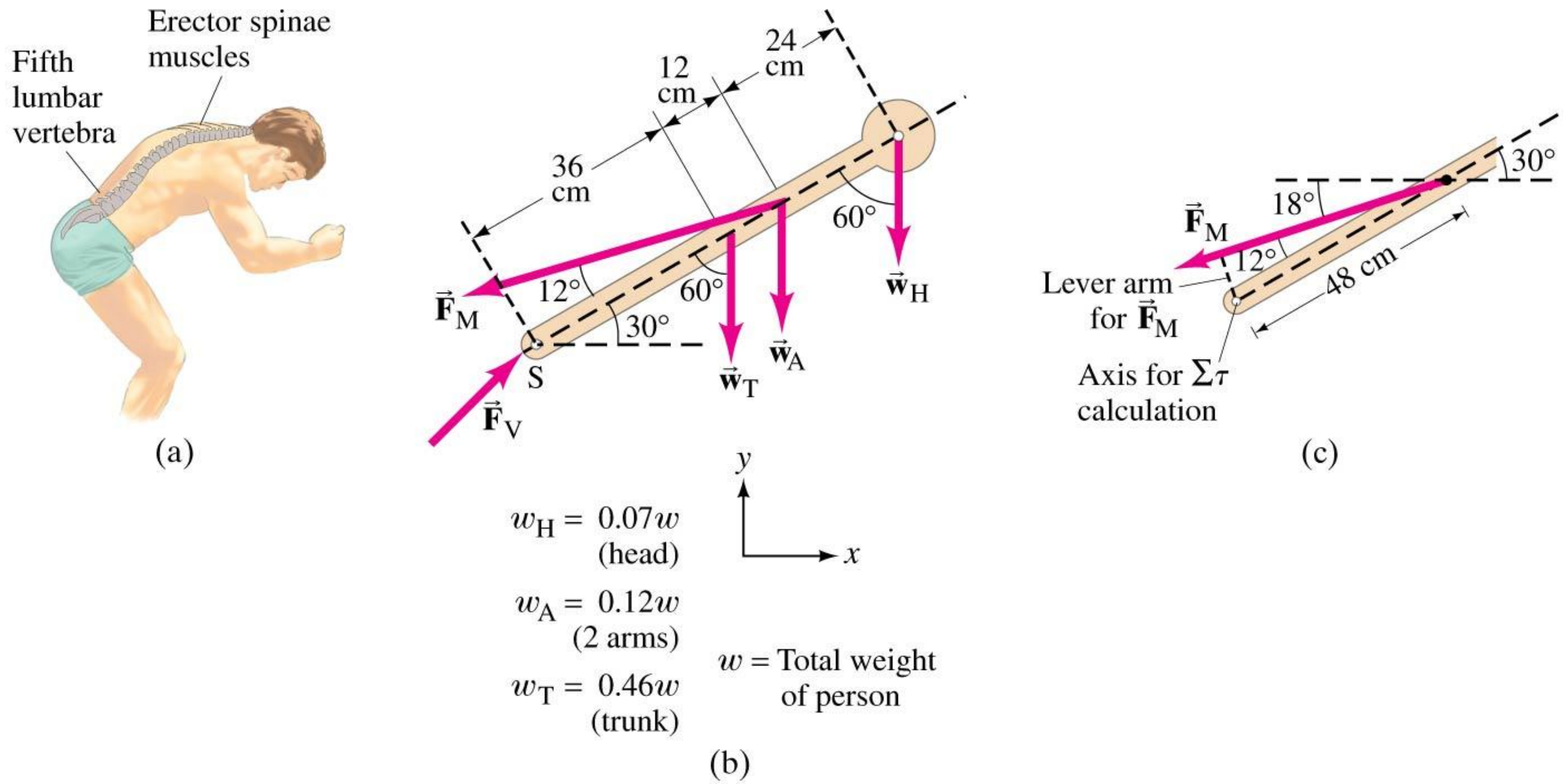
We solve for F_M:

$$F_M = \frac{(0.15\ \text{m})(2.0\ \text{kg})g + (0.35\ \text{m})(5.0\ \text{kg})g}{0.050\ \text{m}} = (41\ \text{kg})g = 400\ \text{N}.$$

(*b*) The lever arm, as calculated about the joint, is reduced by the factor cos 45° for all three forces. Our torque equation will look like the one just above, except that each term will have its lever arm reduced by the same factor, which will cancel out. The same result is obtained, $F_M = 400\ \text{N}$.

9-3 Applications to Muscles and Joints

The angle at which this man's back is bent places an enormous force on the disks at the base of his spine, as the lever arm for F_M is so small.



EXAMPLE 9–9 **Forces on your back.** Calculate the magnitude and direction of the force $\vec{\mathbf{F}}_V$ acting on the fifth lumbar vertebra as represented in Fig. 9–14b.

APPROACH We use the model of the upper body described above and shown in Fig. 9–14b. We can calculate F_M using the torque equation if we take the axis at the base of the spine (point S); with this choice, the other unknown, F_V, doesn't appear in the equation because its lever arm is zero. To figure the lever arms, we need to use trigonometric functions.

SOLUTION For $\vec{\mathbf{F}}_M$, the lever arm (perpendicular distance from axis to line of action of the force) will be the real distance to where the force acts (48 cm) multiplied by sin 12°, as shown in Fig. 9–14c. The lever arms for $\vec{\mathbf{w}}_H$, $\vec{\mathbf{w}}_A$, and $\vec{\mathbf{w}}_T$ can be seen from Fig. 9–14b to be their respective distances from S times sin 60°. F_M tends to rotate the trunk counterclockwise, which we take to be positive. Then $\vec{\mathbf{w}}_H$, $\vec{\mathbf{w}}_A$, $\vec{\mathbf{w}}_T$ will contribute negative torques. Thus $\Sigma\tau = 0$ gives

$$(0.48\,\mathrm{m})(\sin 12^\circ)(F_M) - (0.72\,\mathrm{m})(\sin 60^\circ)(w_H) - (0.48\,\mathrm{m})(\sin 60^\circ)(w_A) - (0.36\,\mathrm{m})(\sin 60^\circ)(w_T) = 0.$$

Solving for F_M and putting in the values for w_H, w_A, w_T given in Fig. 9–14b, we find

$$F_M = \frac{(0.72\,\mathrm{m})(0.07w) + (0.48\,\mathrm{m})(0.12w) + (0.36\,\mathrm{m})(0.46w)}{(0.48\,\mathrm{m})(\sin 12^\circ)}(\sin 60^\circ)$$
$$= 2.37w \approx 2.4w,$$

where w is the total weight of the body. To get the components of $\vec{\mathbf{F}}_V$ we use the x and y components of the force equation (noting that $30^\circ - 12^\circ = 18^\circ$):

$$\Sigma F_y = F_{Vy} - F_M \sin 18^\circ - w_H - w_A - w_T = 0$$

so

$$F_{Vy} = 1.38w \approx 1.4w,$$

and

$$\Sigma F_x = F_{Vx} - F_M \cos 18^\circ = 0$$

so

$$F_{Vx} = 2.25w \approx 2.3w,$$

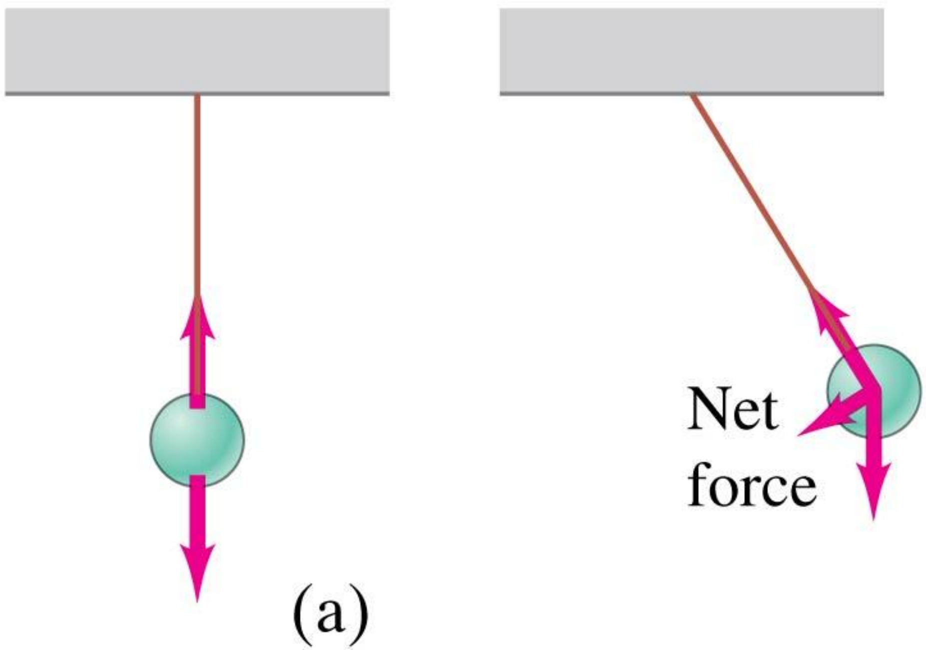
where we keep 3 significant figures for calculating, but round off to 2 for giving the answer. Then

$$F_V = \sqrt{F_{Vx}^2 + F_{Vy}^2} = 2.6w.$$

The angle θ that F_V makes with the horizontal is given by $\tan\theta = F_{Vy}/F_{Vx} = 0.61$, so $\theta = 32^\circ$.

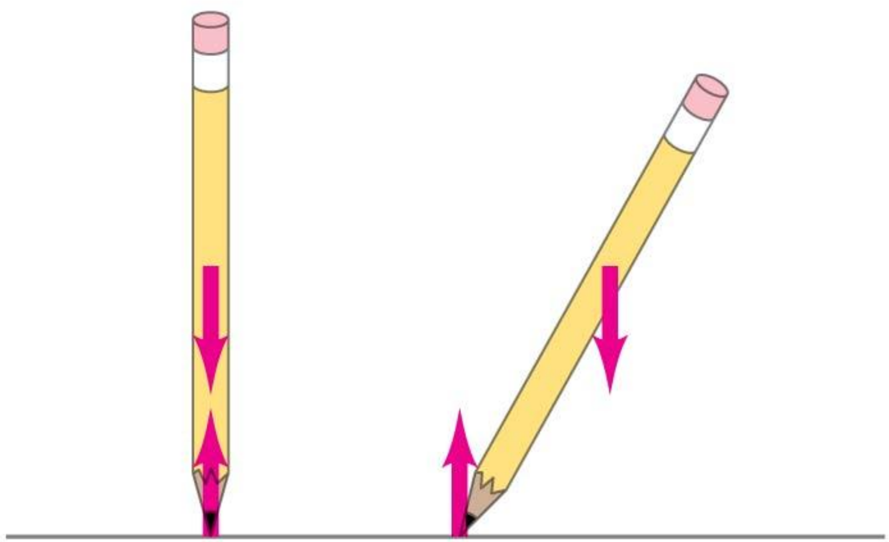
9-4 Stability and Balance

If the forces on an object are such that they tend to return it to its equilibrium position, it is said to be in stable equilibrium.



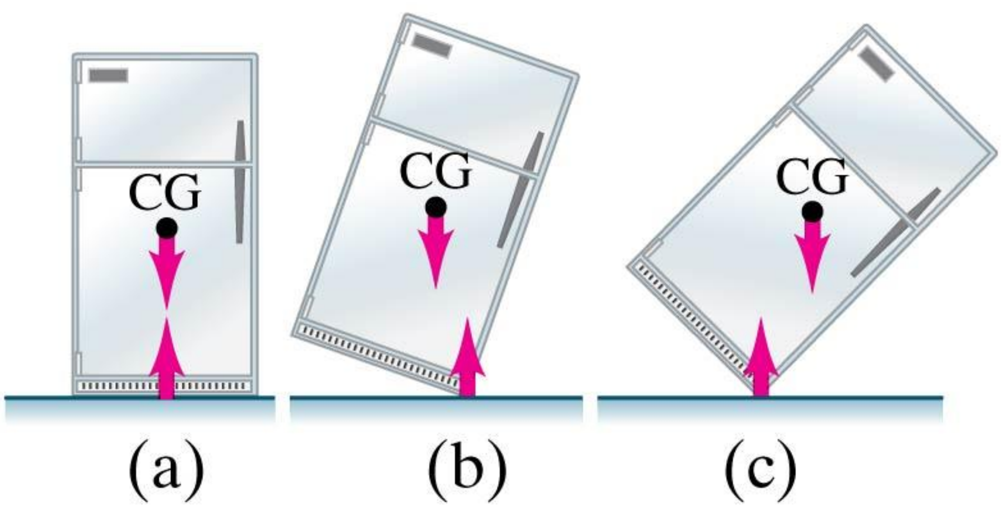
9-4 Stability and Balance

If, however, the forces tend to move it away from its equilibrium point, it is said to be in unstable equilibrium.


(b)

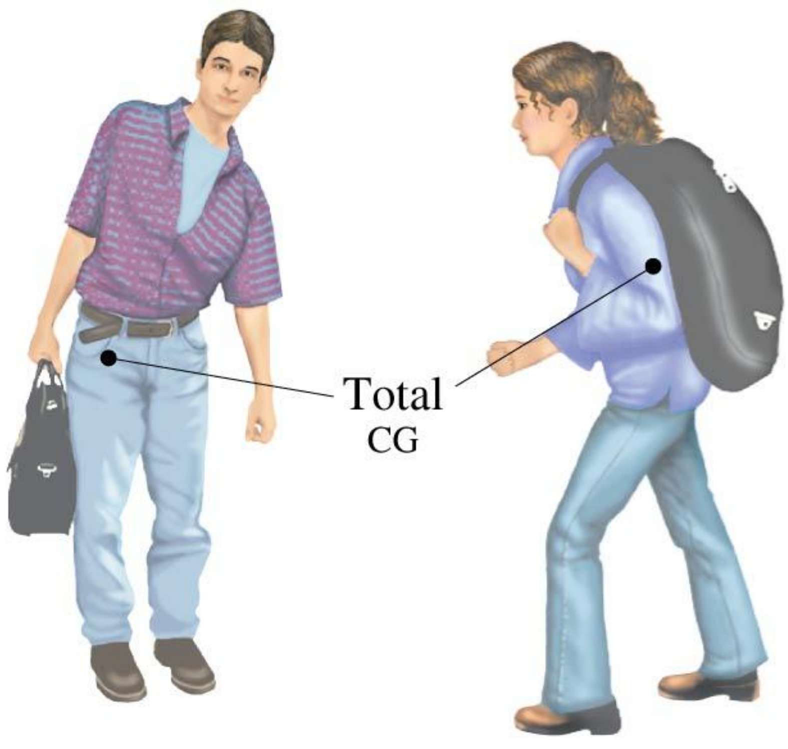
9-4 Stability and Balance

An object in stable equilibrium may become unstable if it is tipped so that its center of gravity is outside the pivot point. Of course, it will be stable again once it lands!



9-4 Stability and Balance

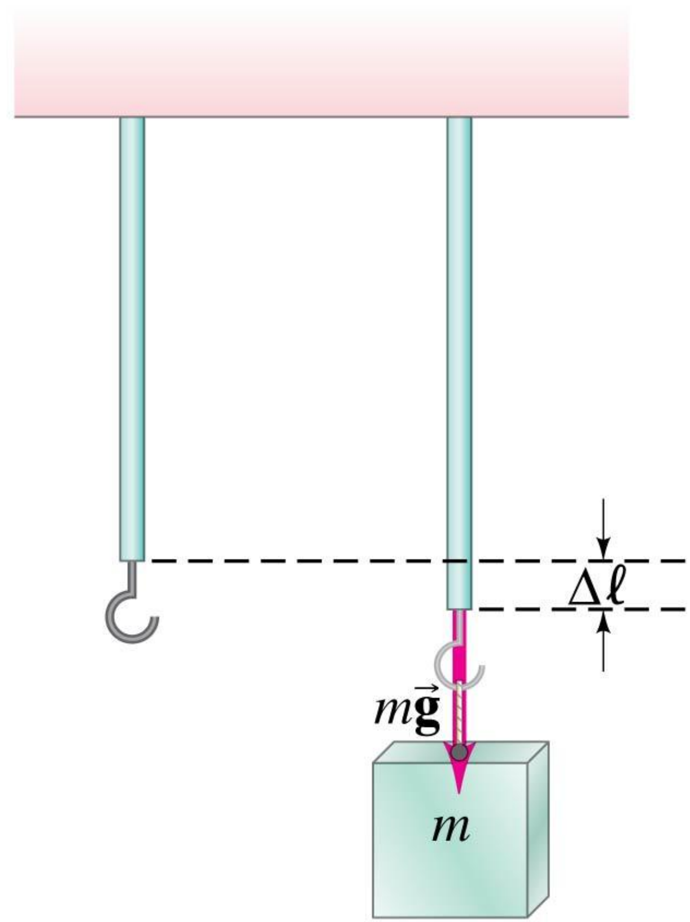
People carrying heavy loads automatically adjust their posture so their center of mass is over their feet. This can lead to injury if the contortion is too great.



9-5 Elasticity; Stress and Strain

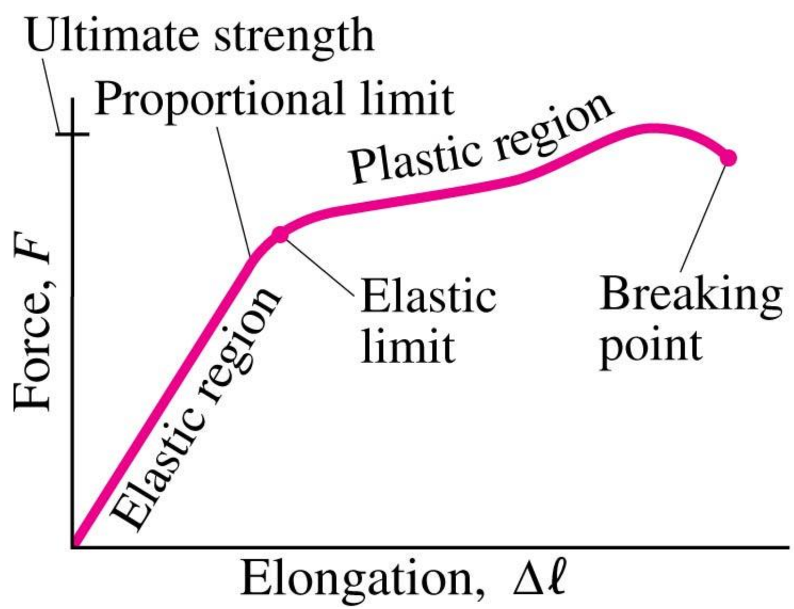
Hooke's law: the change in length is proportional to the applied force.

$$F = k\,\Delta\ell \qquad (9\text{-}3)$$



9-5 Elasticity; Stress and Strain

This proportionality holds until the force reaches the proportional limit. Beyond that, the object will still return to its original shape up to the elastic limit. Beyond the elastic limit, the material is permanently deformed, and it breaks at the breaking point.



9-5 Elasticity; Stress and Strain

The change in length of a stretched object depends not only on the applied force, but also on its length and cross-sectional area, and the material from which it is made.

The material factor is called Young's modulus, and it has been measured for many materials.

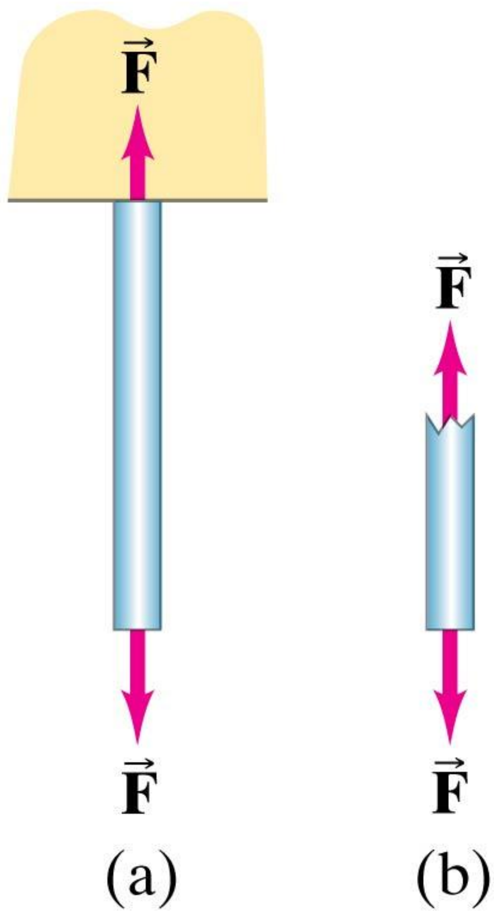
$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

The Young's modulus is then the stress divided by the strain.

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta \ell}{\ell_0}$$

9-5 Elasticity; Stress and Strain

In tensile stress, forces tend to stretch the object.

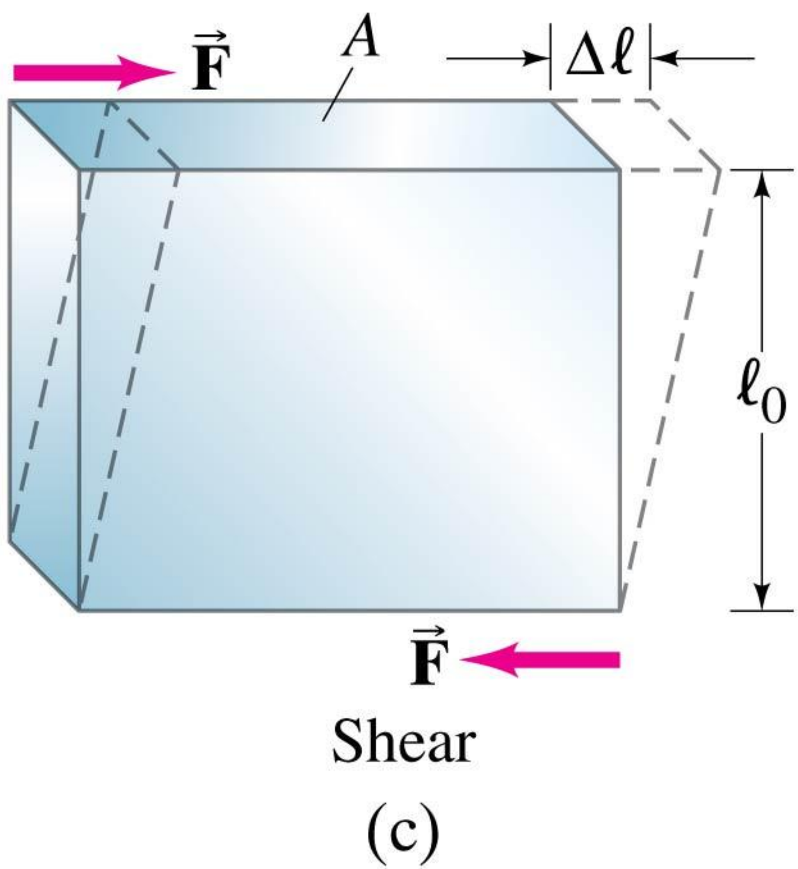


9-5 Elasticity; Stress and Strain

Compressional stress is exactly the opposite of tensional stress. These columns are under compression.

9-5 Elasticity; Stress and Strain

Shear stress tends to deform an object:



EXAMPLE 9-10

Tension in piano wire. A 1.60-m-long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.25 cm when tightened?

APPROACH We assume Hooke's law holds, and use it in the form of Eq. 9–4, finding E for steel in Table 9–1.

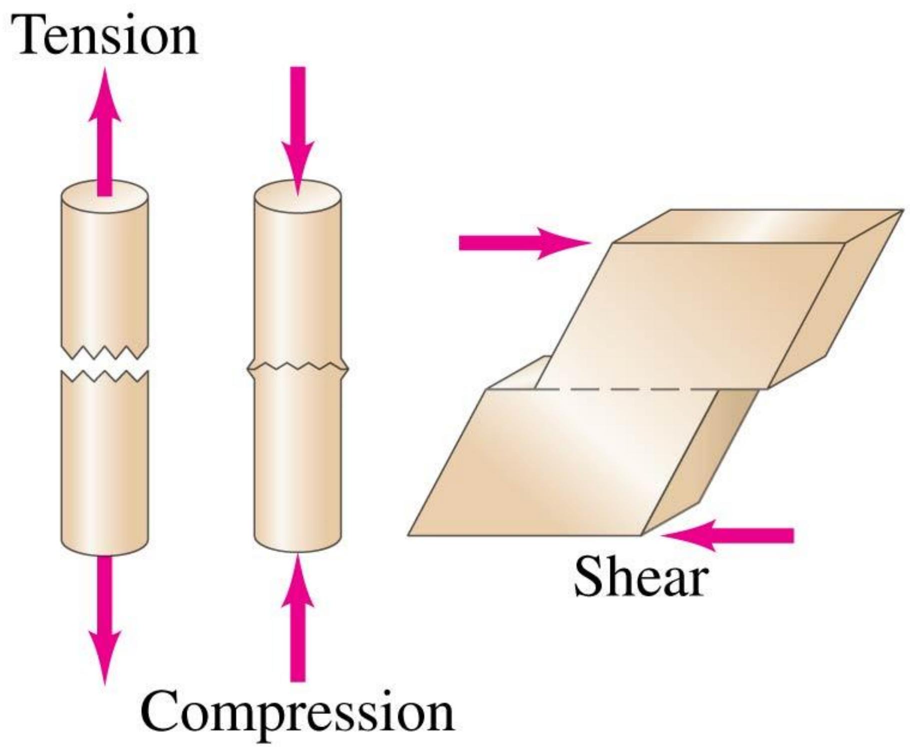
SOLUTION We solve for F in Eq. 9–4 and note that the area of the wire is $A = \pi r^2 = (3.14)(0.0010\,\mathrm{m})^2 = 3.14 \times 10^{-6}\,\mathrm{m}^2$. Then

$$\begin{aligned} F &= E\frac{\Delta \ell}{\ell_0}A \\ &= \left(2.0 \times 10^{11}\,\mathrm{N/m^2}\right)\left(\frac{0.0025\,\mathrm{m}}{1.60\,\mathrm{m}}\right)\left(3.14 \times 10^{-6}\,\mathrm{m^2}\right) \\ &= 980\,\mathrm{N}. \end{aligned}$$

9-6 Fracture

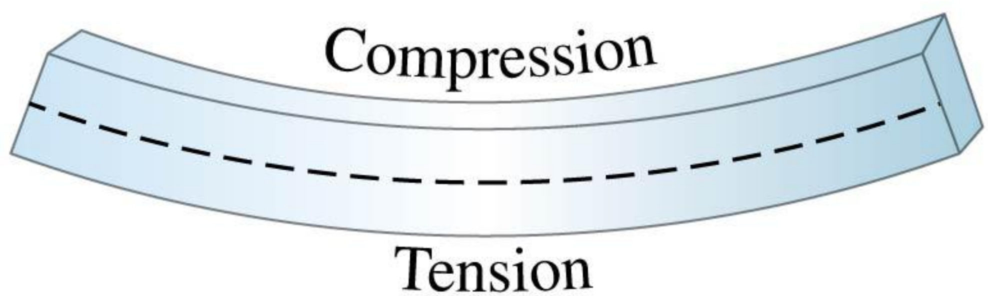
If the stress is too great, the object will fracture. The ultimate strengths of materials under tensile stress, compressional stress, and shear stress have been measured.

When designing a structure, it is a good idea to keep anticipated stresses less than 1/3 to 1/10 of the ultimate strength.



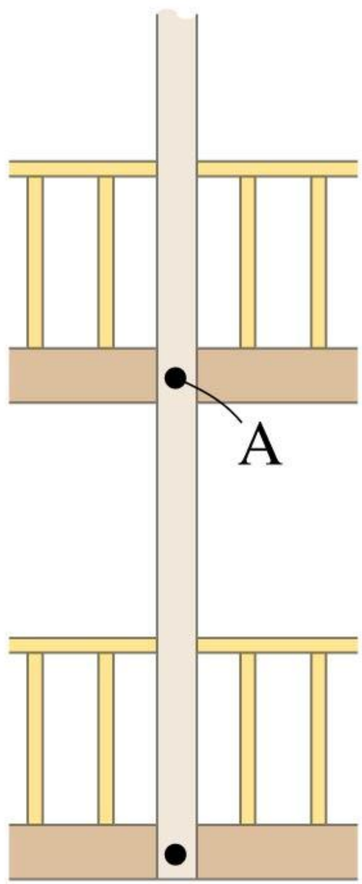
9-6 Fracture

A horizontal beam will be under both tensile and compressive stress due to its own weight.



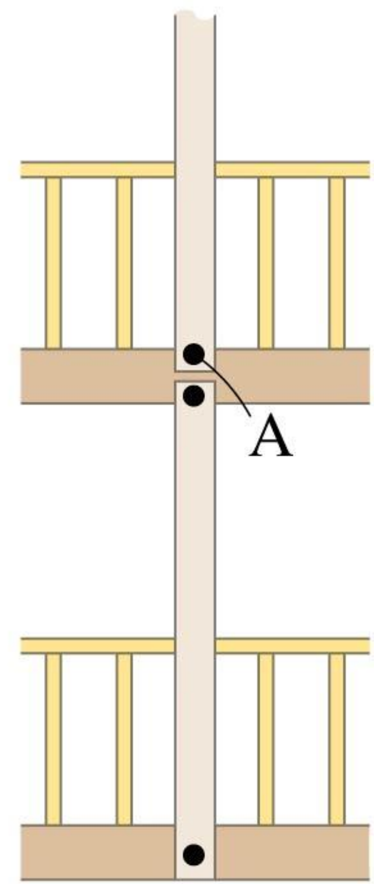
9-6 Fracture

Here is the original design of a walkway. The central supports were to be 14 meters long.



(a)

During installation, it was decided that the long supports were too difficult to install; the walkways were installed this way instead.

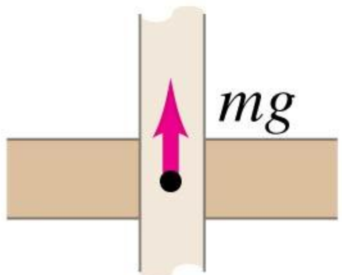


(b)

9-6 Fracture

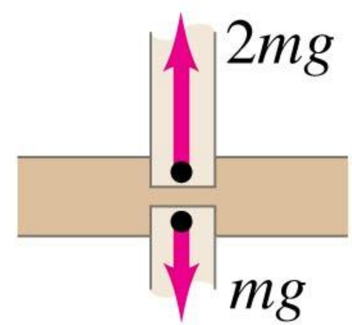
The change does not appear major until you look at the forces on the bolts:

The net force on the pin in the original design is *mg*, upwards.



(c) Force on pin A exerted by vertical rod

When modified, the net force on both pins together is still *mg*, but the top pin has a force of 2*mg* on it— enough to cause it to fail, which it did.



(d) Forces on pins at A exerted by vertical rods

EXAMPLE 9-11

Breaking the piano wire. The steel piano wire we discussed in Example 9–10 was 1.60 m long with a diameter of 0.20 cm. Approximately what tension force would break it?

APPROACH We set the tensile stress F/A equal to the ultimate tensile strength of steel given in Table 9–2, and we choose the highest value which represents high-carbon steel.

SOLUTION The wire's area is $A = \pi r^2$, where $r = 0.10\,\text{cm} = 1.0 \times 10^{-3}\,\text{m}$. Table 9–2 tells us

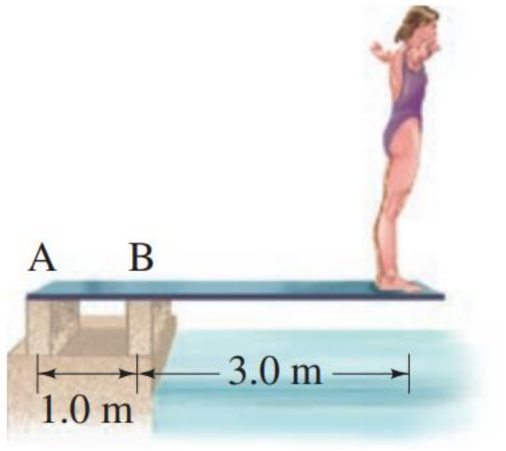
$$\frac{F}{A} = 2500 \times 10^6\,\text{N/m}^2,$$

so the wire would likely break if the force exceeded

$$F = (2500 \times 10^6\,\text{N/m}^2)(\pi)(1.0 \times 10^{-3}\,\text{m})^2 = 8000\,\text{N}.$$

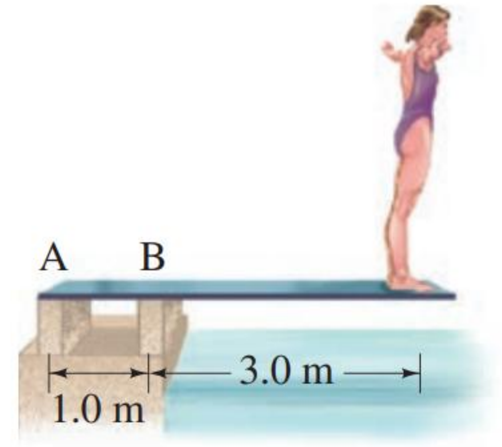
Problem 4:

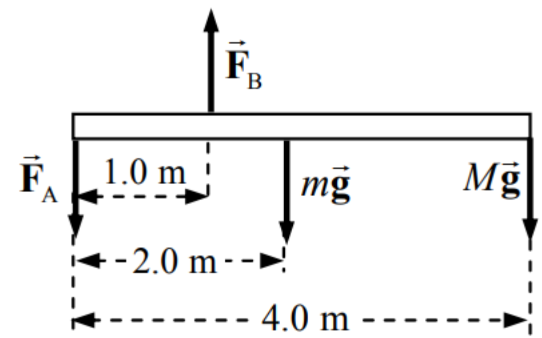
What is the mass of the diver in Fig. 9–49 if she exerts a torque of 1800 N.m on the board, relative to the left (A) support post?



Problem 5:

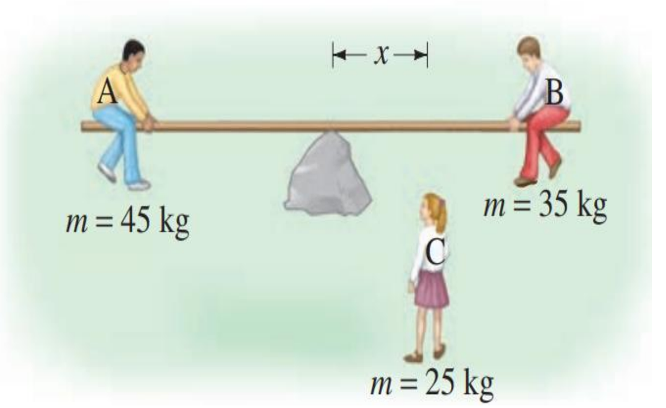
(II) Calculate the forces F_A and F_B that the supports exert on the diving board of Fig. 9–49 when a 52-kg person stands at its tip. (a) Ignore the weight of the board. (b) Take into account the board's mass of 28 kg. Assume the board's CG is at its center.



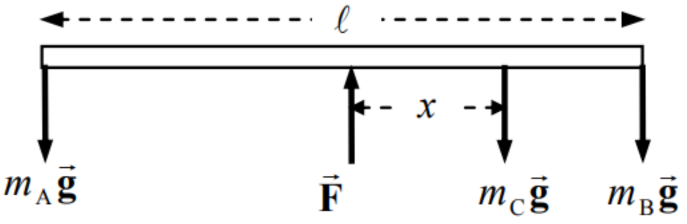


Problem 17:

Three children are trying to balance on a seesaw, which includes a fulcrum rock acting as a pivot at the center, and a very light board 3.2 m long (Fig. 9–57). Two playmates are already on either end. Boy A has a mass of 45 kg, and boy B a mass of 35 kg. Where should girl C, whose mass is 25 kg, place herself so as to balance the seesaw?

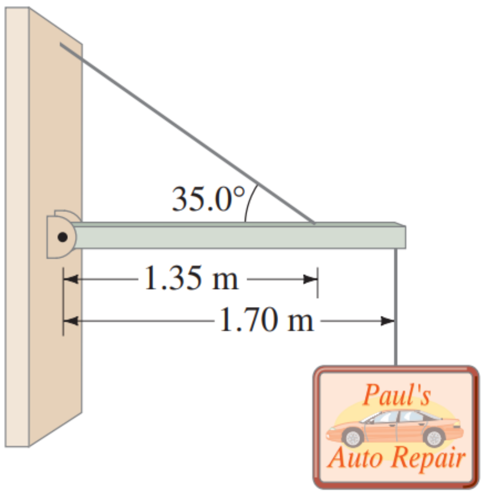


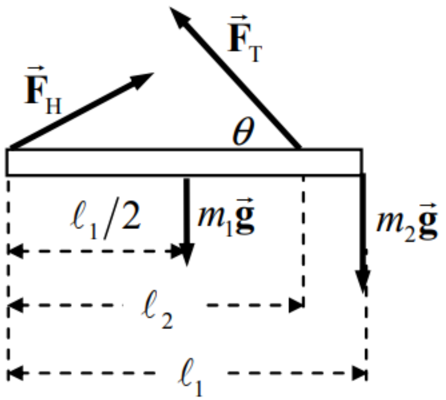
Solution:



Problem 18:

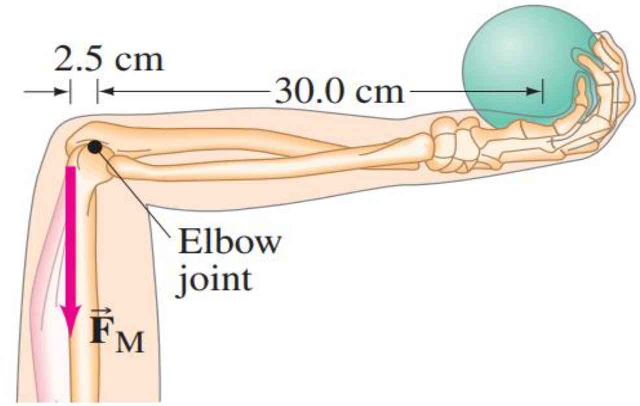
A shop sign weighing 215 N hangs from the end of a uniform 155-N beam as shown in Fig. 9–58. Find the tension in the supporting wire (at 35.0°), and the horizontal and vertical forces exerted by the hinge on the beam at the wall.

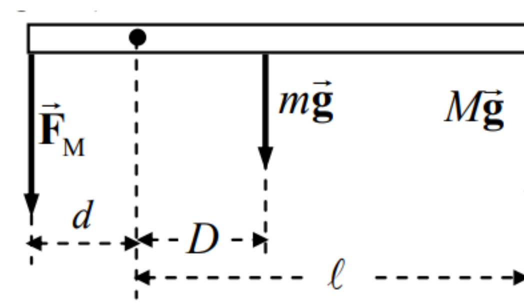




Problem 32:

Approximately what magnitude force, F_M, must the extensor muscle in the upper arm exert on the lower arm to hold a 7.3-kg shot put (Fig. 9–70)? Assume the lower arm has a mass of 2.3 kg and its CG is 12.0 cm from the elbow-joint pivot.





Problem 38 & 39:

A marble column of cross-sectional area 1.4 m^2 supports a mass of 25,000 kg. (a) What is the stress within the column? (b) What is the strain?

By how much is the column in Problem 38 shortened if it is 8.6 m high?

Problem 43:

A 15-cm-long tendon was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm. Calculate Young's modulus of this tendon.

Problem 46:

The femur bone in the human leg has a minimum effective cross section of about 3.0 cm^2. How much compressive force can it withstand before breaking?