# Chapter 7

# Center of Mass (CM)

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# Section 7-8: Center of Mass

- Up to now, we've been mainly concerned with the motion of single (point) particles.
- To treat extended objects, we've implicitly approximated the object as a point particle & treated it ***as if*** it had all of its mass at a point!

we treat it as if it had all its mass at a point

***How is this possible?***

- Real, extended objects have complex motion, including (possibly): translation, rotation, & vibration!

- ***Experimental Observation*:** No matter how extended an object is, there is one point in the object

(≡ **the Center of Mass**)

that moves in the same path that a point particle with the same mass as the object would move (subject to the same forces).

- ***Center of Mass:*** That point in an object which, as far as Newton's Laws are concerned, moves as ***if all*** of it's mass were concentrated at that point
  - Proof from Newton's Laws: See Section 7-10 (not covered in class!).

# Example: Diver Motion Complicated!

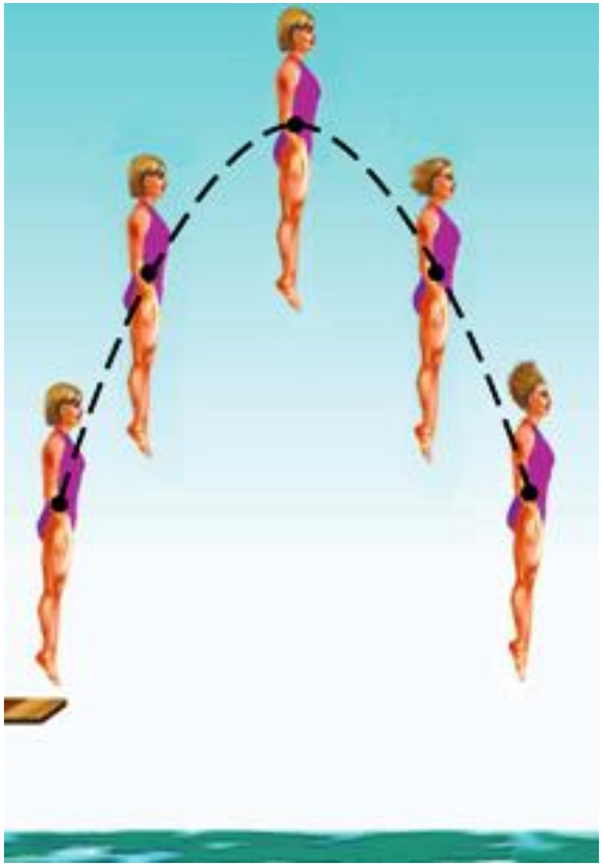


Figure (a)
A diver moving with pure translation.

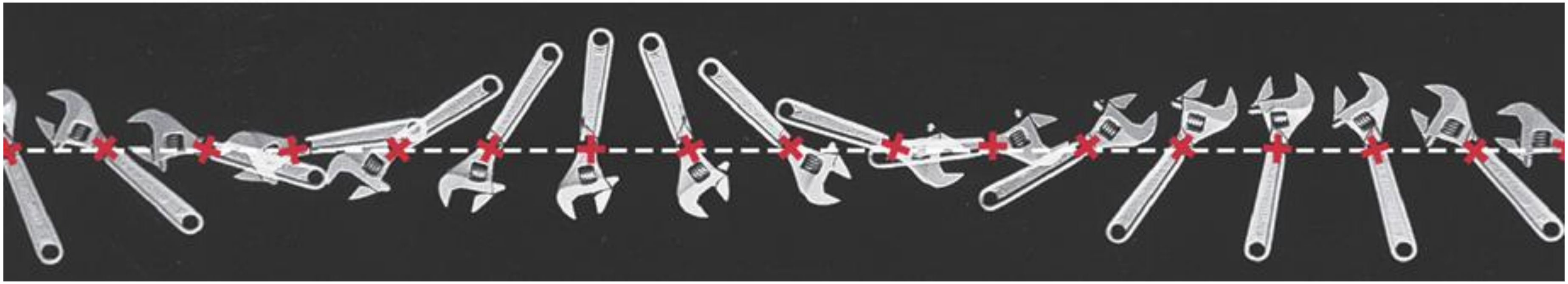
No matter what types of motion she does, there is always one point that moves in the same path a particle would take if subjected to the same force as the diver. This point is called her center of mass (CM).

**Her CM motion is the same as a projectile (a parabola!)**

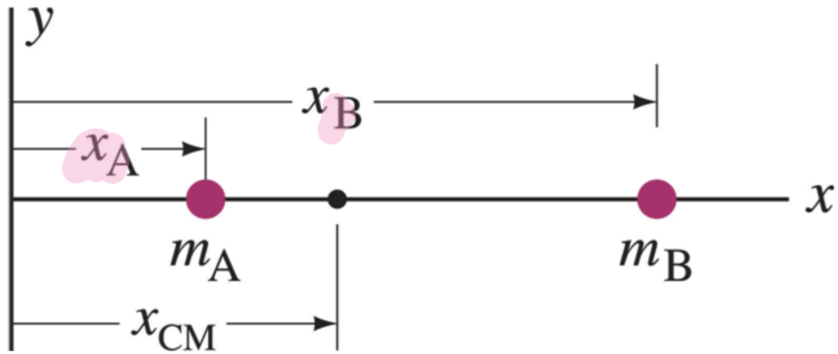
Figure (b)
A diver moving with translation + rotation.

# Example: A Wrench is Thrown Complicated!

The <u>**general motion**</u> of an object can be considered as the **sum of the translational motion of the CM, plus rotational, or other forms of motion about the CM.**



*Assume that* objects are made up of a large number of tiny point particle masses. Consider 2 point masses $\mathbf{m_A}$ & $\mathbf{m_B}$ in 1 dimension, along the **x**-axis, at




positions $\mathbf{x_A}$ & $\mathbf{x_B}$ as in the figure. The **Center of Mass** **(CM)** of these two point masses is ***DEFINED*** as

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M}$$

$\mathbf{M = m_A + m_B}$ = total mass. Extend to 2 dimensions & the **y** axis:

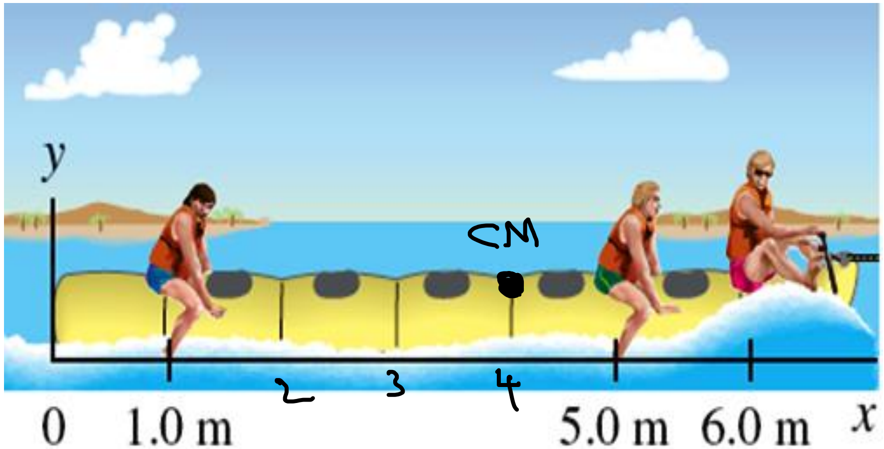
$$\mathbf{y_{CM} = (m_A y_A + m_B y_B)/(m_A + m_B)}$$

Extend to more than two masses (say, 3 masses):

$$\mathbf{x_{CM} = (m_A x_A + m_B x_B + m_C x_C)/(m_A + m_B + m_C)}$$

# Example 7-12: CM of three guys on a raft.

Three people of roughly equal mass **m** on a lightweight (air-filled) "banana boat" sit along the **x** axis at positions $\mathbf{x_A = 1.0\ m}$, $\mathbf{x_B = 5.0\ m}$, & $\mathbf{x_C = 6.0\ m}$, measured from the left end. Find the CM position. Ignore the boat's mass.




**Solution:**

$$\frac{m(1) + 5m + 6m}{3m} \qquad \frac{12m}{3m} = 4$$

$$x_{CM} = \frac{mx_A + mx_B + mx_C}{m + m + m} = \frac{m(x_A + x_B + x_C)}{3m}$$

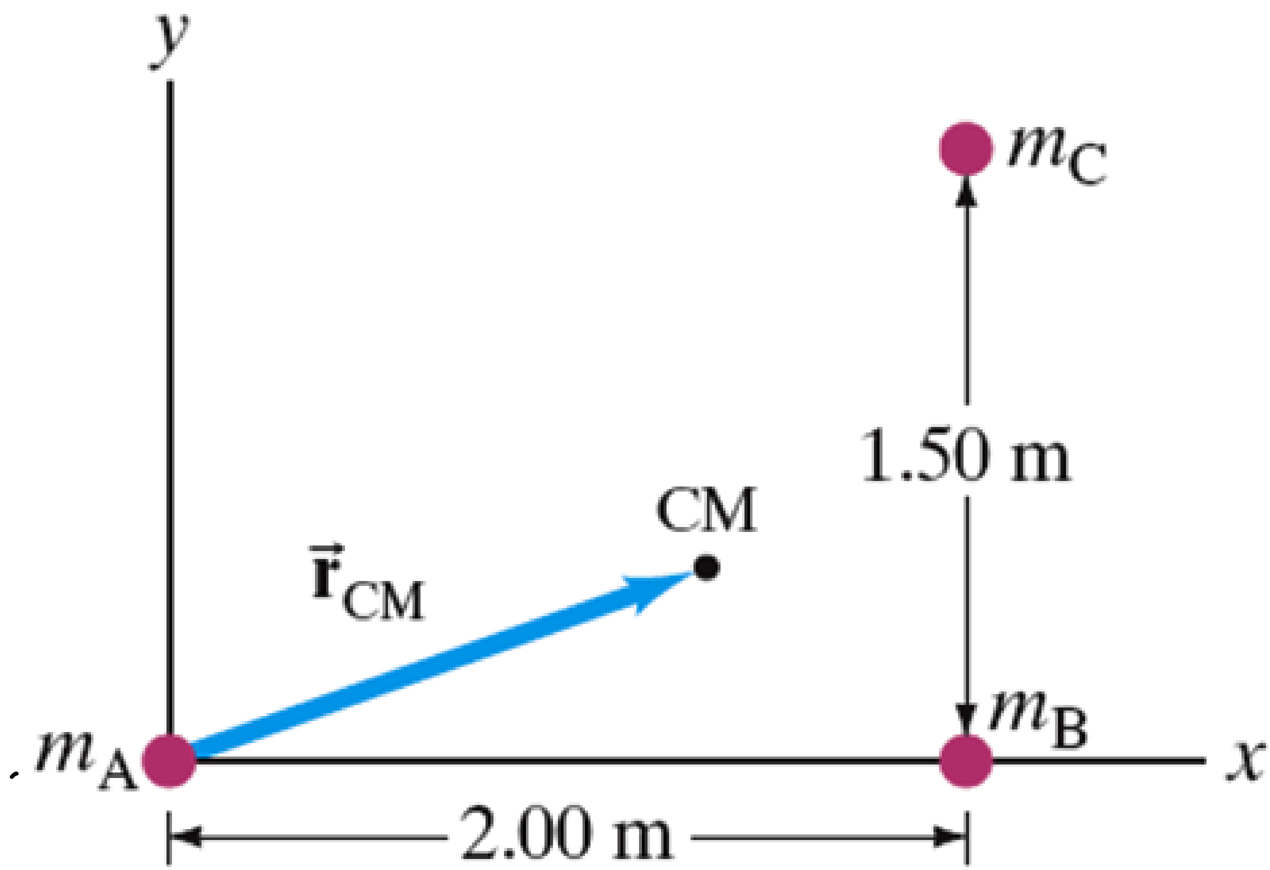
$$= \frac{(1.0\,\text{m} + 5.0\,\text{m} + 6.0\,\text{m})}{3} = \frac{12.0\,\text{m}}{3} = 4.0\,\text{m}.$$

1.3

0.5

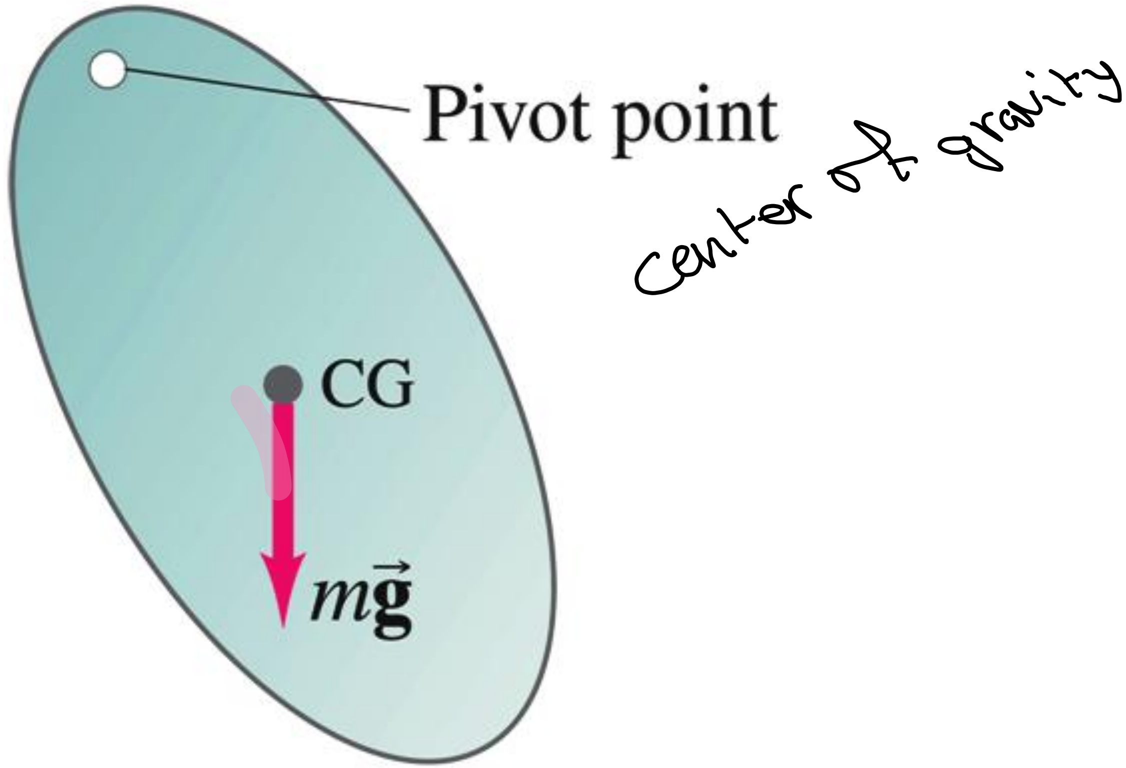
# Example: Three particles in 2-D.

Three particles, each of mass 2.50 kg, are located at the corners of a right triangle whose sides are 2.00 m and 1.50 m long, as shown. Locate the center of mass.

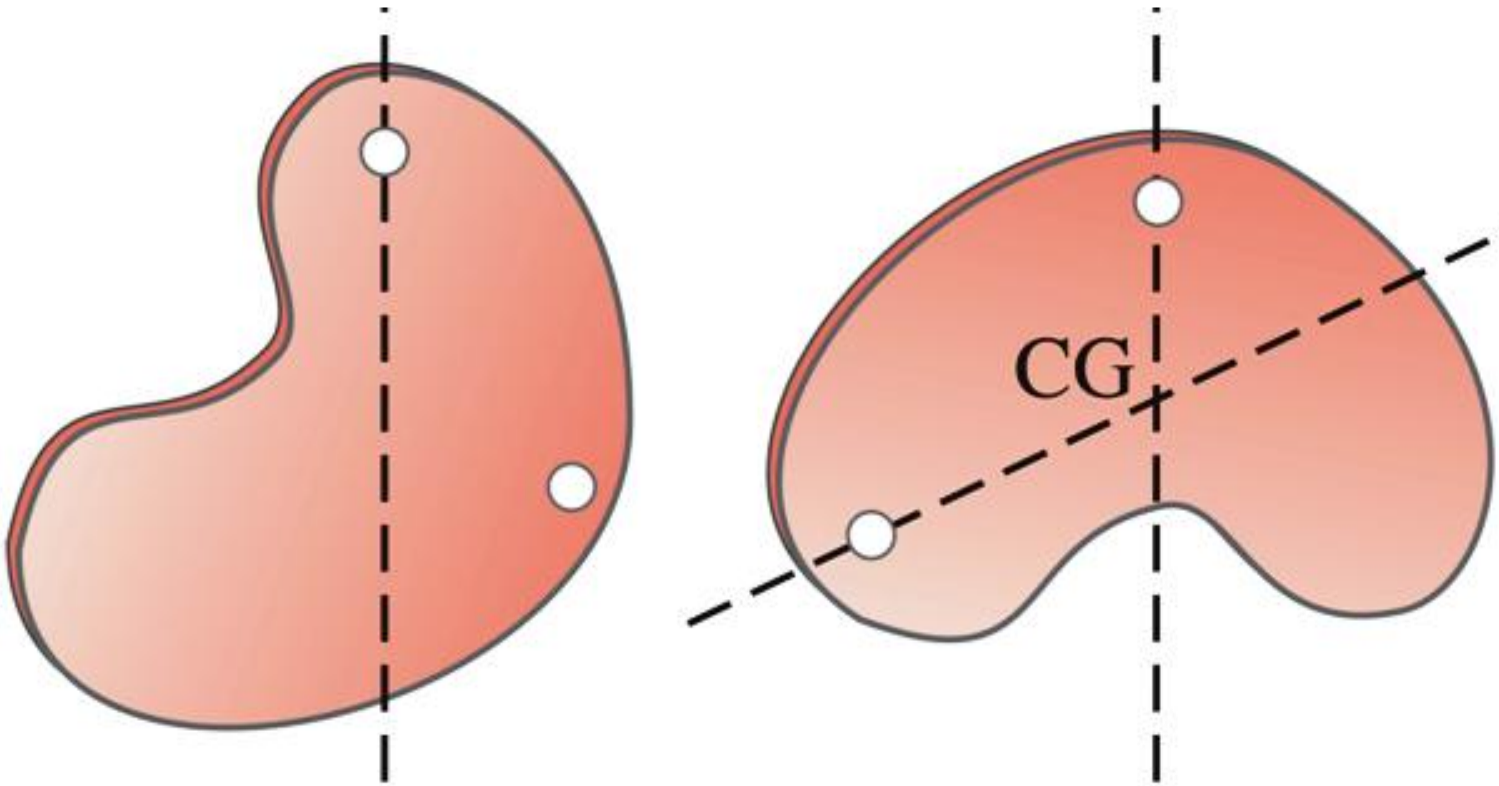


For any object, the **Center of Gravity** is the point where the gravitational force can be considered to act. For cases of interest in this course,

***It is the same as the Center of Mass.***



The **Center of Gravity** can be found experimentally by suspending an object from different points. The CM need not be within the actual object – a doughnut's CM is in the center of the hole.

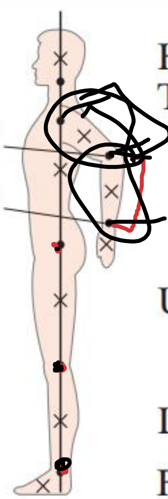


# Sect. 7-9 CM for the Human Body

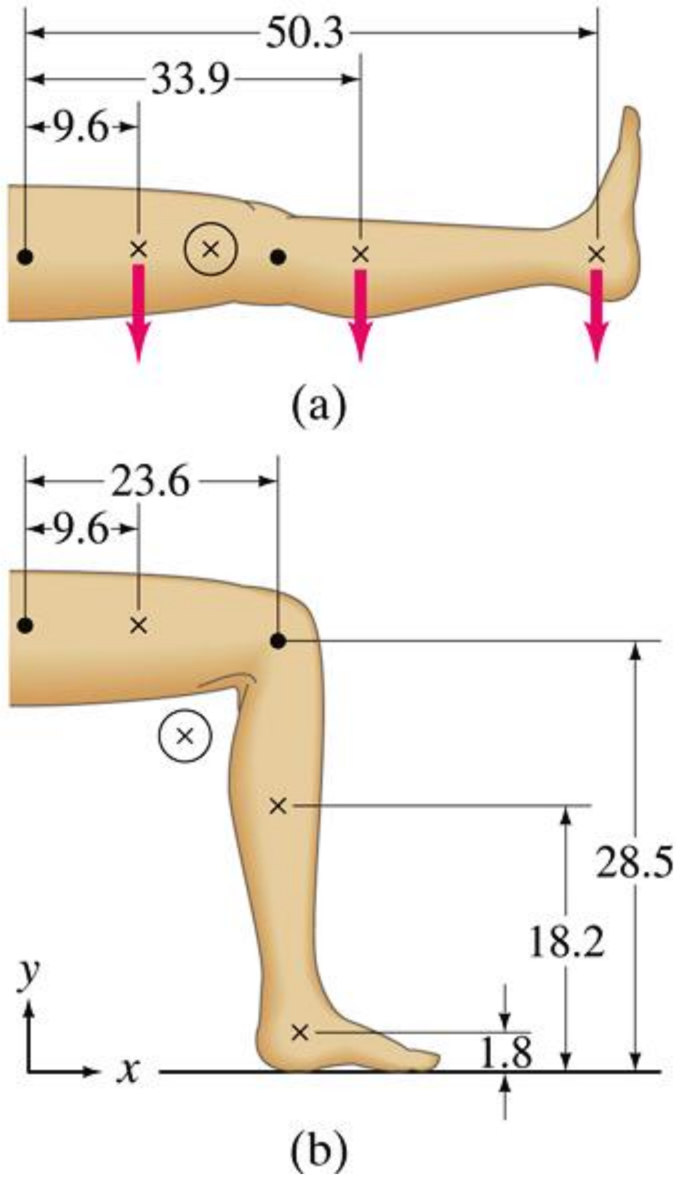
The x’s in the small diagram mark the CM of the listed body segments.

**TABLE 7–1 Center of Mass of Parts of Typical Human Body, given as %**
**(full height and mass = 100 units)**

| Distance of Hinge Points from Floor (%) | Hinge Points (•) (Joints) | Center of Mass (×) (% Height Above Floor) | | Percent Mass |
|---|---|---|---|---|
| 91.2% | Base of skull on spine | Head | 93.5% | 6.9% |
| 81.2% | Shoulder joint | Trunk and neck | 71.1% | 46.1% |
| | elbow 62.2%‡ | Upper arms | 71.7% | 6.6% |
| | wrist 46.2%‡ | Lower arms | 55.3% | 4.2% |
| 52.1% | Hip joint | Hands | 43.1% | 1.7% |
| | | Upper legs (thighs) | 42.5% | 21.5% |
| 28.5% | Knee joint | | | |
| | | Lower legs | 18.2% | 9.6% |
| 4.0% | Ankle joint | Feet | 1.8% | 3.4% |
| | | Body CM = | 58.0% | 100.0% |



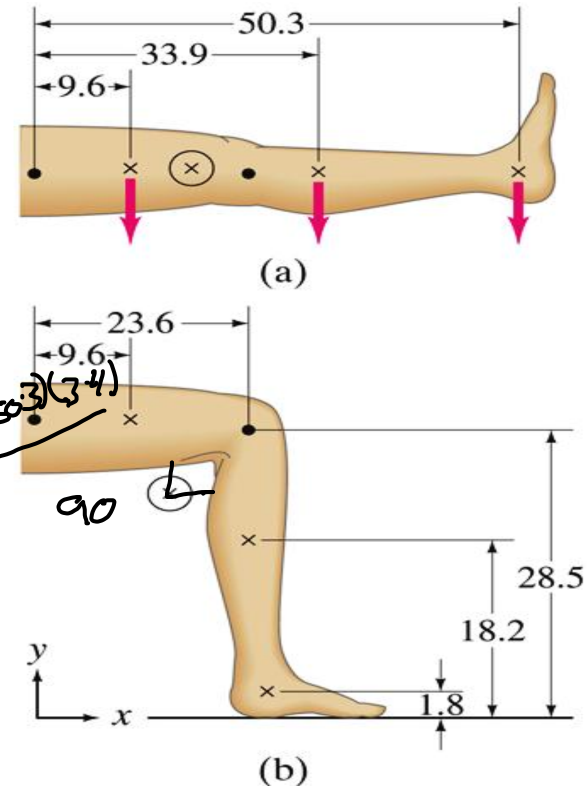
‡ For arm hanging vertically.




The location of the center of mass of the leg (circled) will depend on the position of the leg.

# EXAMPLE 7-13

A leg's CM. Determine the position of the CM of a whole leg (a) when stretched out, and (b) when bent at 90°. See Fig. 7–26. Assume the person is 1.70 m tall




**SOLUTION** (*a*) We determine the distances from the hip joint using Table 7–1 and obtain the numbers (%) shown in Fig. 7–26a. Using Eq. 7–9a, we obtain ($u\ell$ = upper leg, etc.)

$$x_{CM} = \frac{m_{u\ell}x_{u\ell} + m_{\ell\ell}x_{\ell\ell} + m_f x_f}{m_{u\ell} + m_{\ell\ell} + m_f}$$

$$= \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4} = 20.4 \text{ units.}$$

$\frac{9.6(21.5) + (33.9)(9.6) + (50.3)(3.4)}{21.5 + 9.6 + 3.4}$ 90

Thus, the center of mass of the leg and foot is 20.4 units from the hip joint, or $52.1 - 20.4 = 31.7$ units from the base of the foot. Since the person is 1.70 m tall, this is $(1.70\text{ m})(31.7/100) = 0.54\text{ m}$ above the bottom of the foot.

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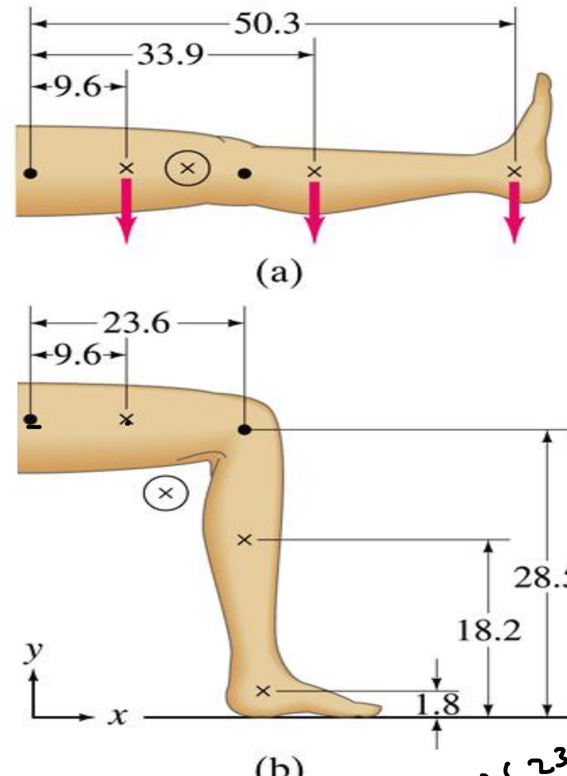


(*b*) We use an *xy* coordinate system, as shown in Fig. 7–26b. First, we calculate how far to the right of the hip joint the CM lies, accounting for all three parts:

$$x_{\mathrm{CM}} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{21.5 + 9.6 + 3.4} = 14.9 \text{ units.}$$

$$y_{cm} = \frac{21.5(28.5) + 9.6(18.2) + 3.4(1.8)}{21.5 + 9.6 + 3.4}$$

For our 1.70-m-tall person, this is $(1.70\text{ m})(14.9/100) = 0.25\text{ m}$ from the hip joint. Next, we calculate the distance, $y_{\mathrm{CM}}$, of the CM above the floor:

$$x_{cm} = \frac{21.5(9.6) + 18.2(23.6) + (3.4)(23.6)}{3.4 + 21.5 + 18.2} = 16.62$$

$$y_{\mathrm{CM}} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{3.4 + 9.6 + 21.5} = 23.0 \text{ units,}$$

or $(1.70\text{ m})(23.0/100) = 0.39\text{ m}$. Thus, the CM is located 39 cm above the floor and 25 cm to the right of the hip joint.

**NOTE** The CM lies outside the body in (*b*).




High jumpers & pole vaulters have developed a technique where their CM actually passes under the bar as they go over it. This allows them to clear higher bars.

# Problem 46

Find the center of mass of the three-mass system shown in Fig. 7–37 relative to the 1.00-kg mass.

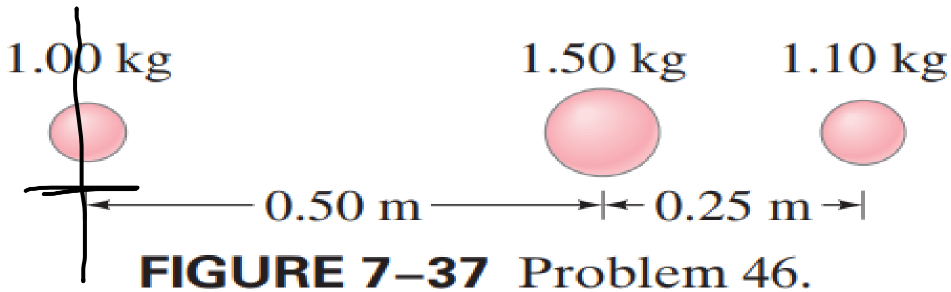


**FIGURE 7–37** Problem 46.

$$x_{CM} = \frac{1(0) + (1.5)(0.5) + (1.1)(0.75)}{3.6} = 0.44$$

## Problem 51

Assume that your proportions are the same as those in Table 7–1, and calculate the mass of one of your legs.

# Problem 53

Use Table 7–1 to calculate the position of the CM of an arm bent at a right angle. Assume that the person is 155 cm tall.