Chapter 6:

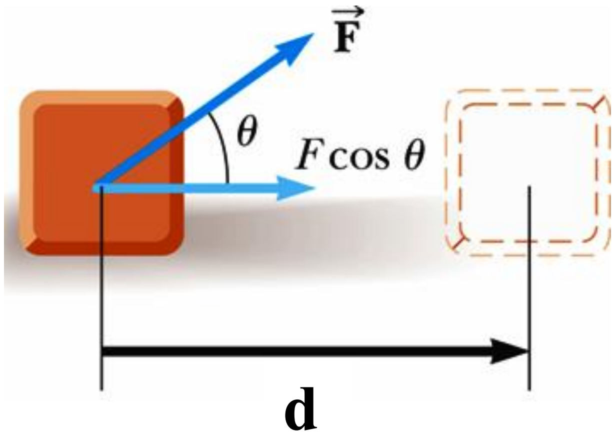
Work and Energy

Department of Physics

The University of Jordan

Sect. 6-1: Work Done by Constant Force

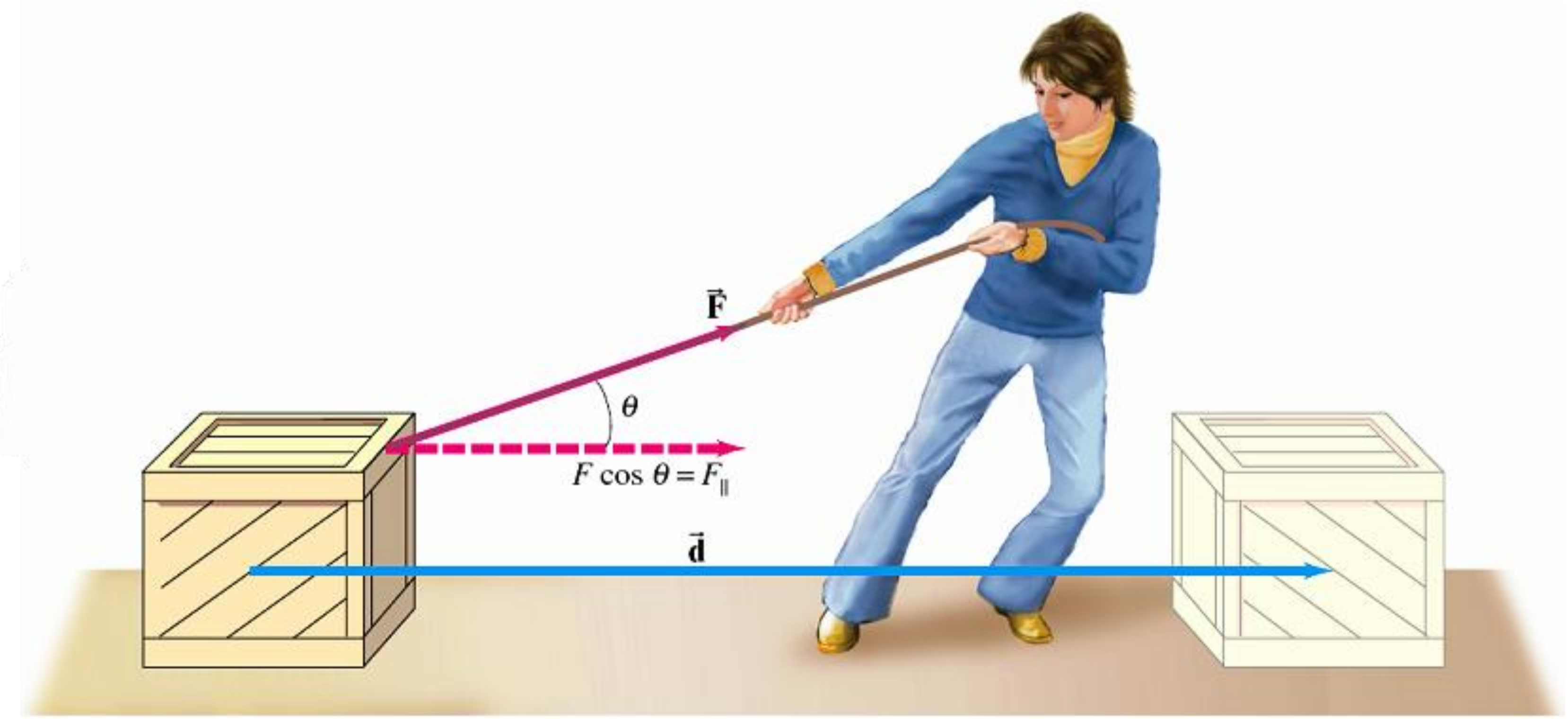
- ***Work:*** Precisely defined in physics. Describes **what is accomplished** by a force in moving an object through a distance.



For an object moving under the influence of a ***Constant Force***, the work done (**W**) ≡ the product of the magnitude of the displacement (**d**) × the component of force parallel to the displacement ($\mathbf{F}_{||}$).

$$\mathbf{W \equiv F_{||}d = Fd\cos\theta}$$

Work Done by a Constant Force



Work

$$W \equiv F_{\|}d = Fd\cos\theta$$

For a ***CONSTANT*** force!

$$\mathbf{W = F_{||}d = Fd\cos\theta}$$

- Consider a simple ***special case*** when **F** & **d** are parallel:

$$\mathbf{\theta = 0, \cos\theta = 1}$$

$\Rightarrow$

$$\mathbf{W = Fd}$$

- Example: **d = 50 m, F = 30 N**

$$\mathbf{W = (30N)(50m) = 1500\ N\ m}$$

- ***Work units*: Newton - meter = Joule**

$$\mathbf{1\ N.m = 1\ Joule = 1\ J}$$

$W = F_{||}d = Fd\cos\theta$

- Can exert a force & do no work!

  Could have $\mathbf{d} = 0 \Rightarrow \mathbf{W} = 0$

  Could have $\mathbf{F} \perp \mathbf{d}$

  $\Rightarrow \theta = 90^o, \cos\theta = 0$

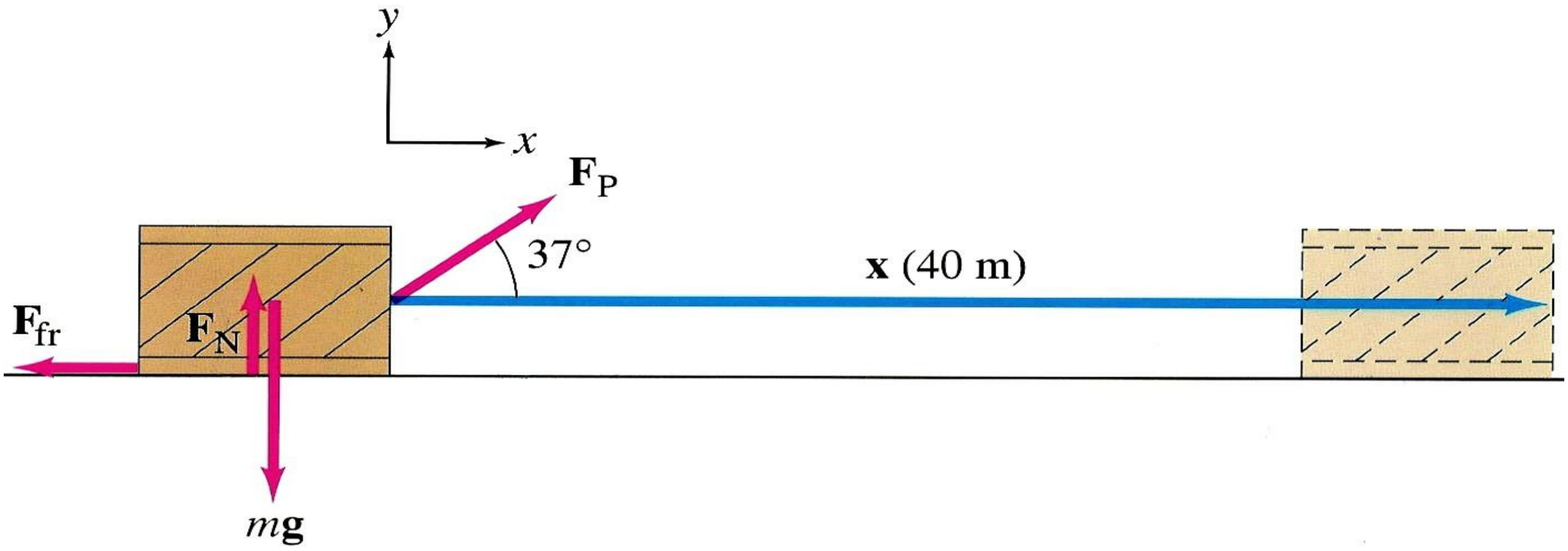
  $\Rightarrow \mathbf{W} = 0$

  Example, walking at constant **v** with a grocery bag:

FIGURE 6–2 Work done on the bag of groceries in this case is zero since **F** is perpendicular to the displacement **d**.

Example 6-1

EXAMPLE 6–1 **Work done on a crate.** A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_P = 100\text{ N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $\vec{\mathbf{F}}_{fr} = 50\text{ N}$. Determine (*a*) the work done by each force acting on the crate, and (*b*) the net work done on the crate.



Example 6-1

Solution:

(a)

$$W_G = mgx\cos 90^\circ = 0$$
$$W_N = F_N x\cos 90^\circ = 0.$$

$$W_P = F_P x\cos\theta = (100\,\text{N})(40\,\text{m})\cos 37^\circ = 3200\,\text{J}.$$

$$W_{fr} = F_{fr} x\cos 180^\circ = (50\,\text{N})(40\,\text{m})(-1) = -2000\,\text{J}.$$

(b)

$$W_{net} = W_G + W_N + W_P + W_{fr}$$
$$= 0 + 0 + 3200\,\text{J} - 2000\,\text{J} = 1200\,\text{J}.$$

or

$$W_{net} = (F_{net})_x x = (F_P\cos\theta - F_{fr})x$$
$$= (100\,\text{N}\cos 37^\circ - 50\,\text{N})(40\,\text{m}) = 1200\,\text{J}.$$

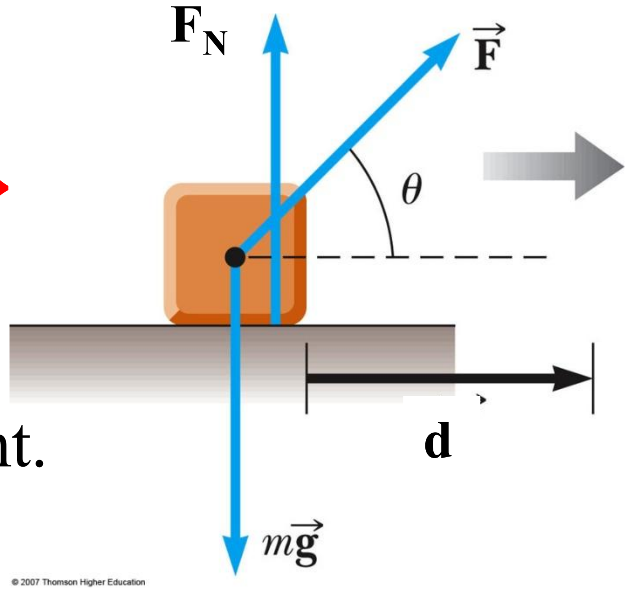
Solving Work Problems

1. **Sketch** a free-body diagram.
2. **Choose** a coordinate system.
3. **Apply** **Newton's Laws** to determine any unknown forces.
4. **Find** the work done by a specific force.
5. **Find** the ***net work*** by either
   a. **Find** the ***net force*** & then find the work it does, or
   b. **Find** the work done by each force & add.

$$W = F_{||}d = Fd \cos\theta$$

A Typical Problem

An object displaced by force **F** on a frictionless, **horizontal surface**. The free body diagram is shown. →



The normal force $\mathbf{F_N}$ & weight **mg** do no work in the process, since both are perpendicular to the displacement.

Angles for forces:

Normal force: θ = 90˚, cosθ = 0

Weight: θ = 270 (or - 90°), cosθ = 0

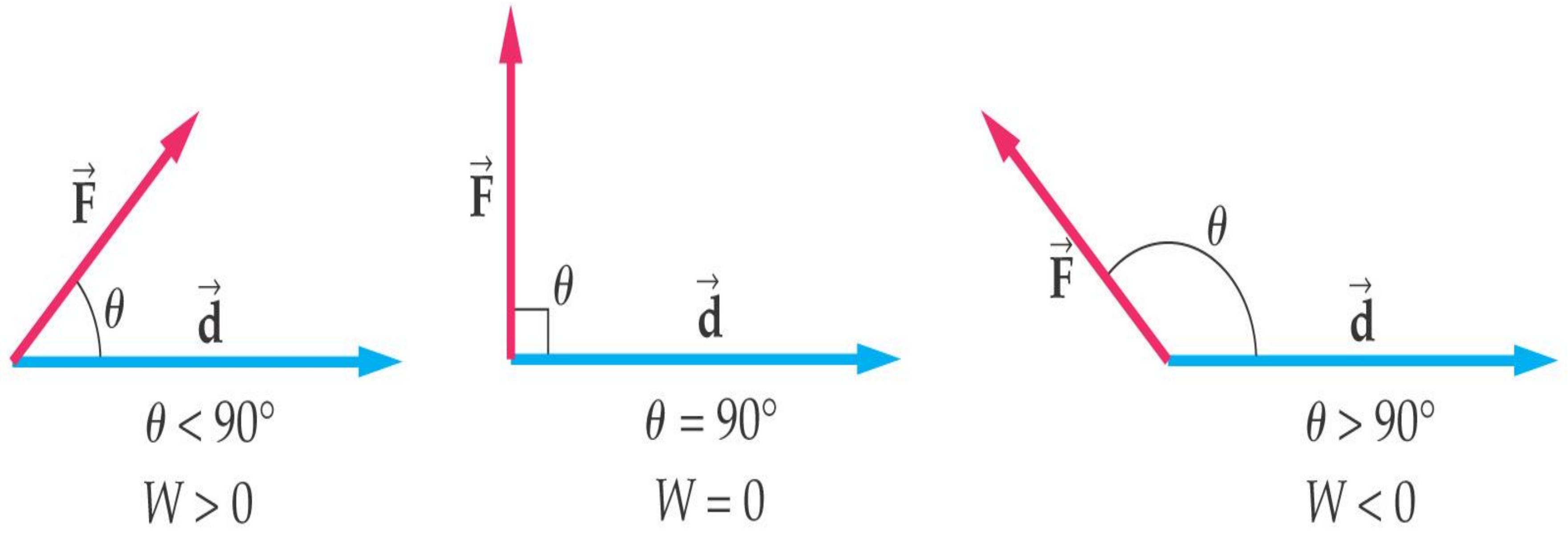
Example

The force shown has magnitude $\mathbf{F_P = 20\ N}$ & makes an angle $\boldsymbol{\theta = 30°}$ to the ground. Calculate the work done by this force when the wagon is dragged a displacement $\mathbf{d = 100\ m}$ along the ground.

$$W = F_{\|}d = Fd\cos\theta = 20 \times 100 \cos 30 = 1732\ J$$

Work Done by a Constant Force

The work done may be positive, zero, or negative, depending on the angle between the force and the displacement:



6.3 Kinetic Energy, and the Work-Energy Principle

- ***Energy***: Traditionally defined as the ability to do work. We now know that not all forces are able to do work; however, we are dealing in these chapters with mechanical energy, which does follow this definition.
- ***Kinetic Energy (KE)*** ≡ The energy of motion

  "Kinetic" ≡ Greek word for motion

  An object in motion has the ability to do work.

$$KE = (½)m(v_2)^2$$

Work-Energy Principle

$$W_{net} = \Delta KE$$

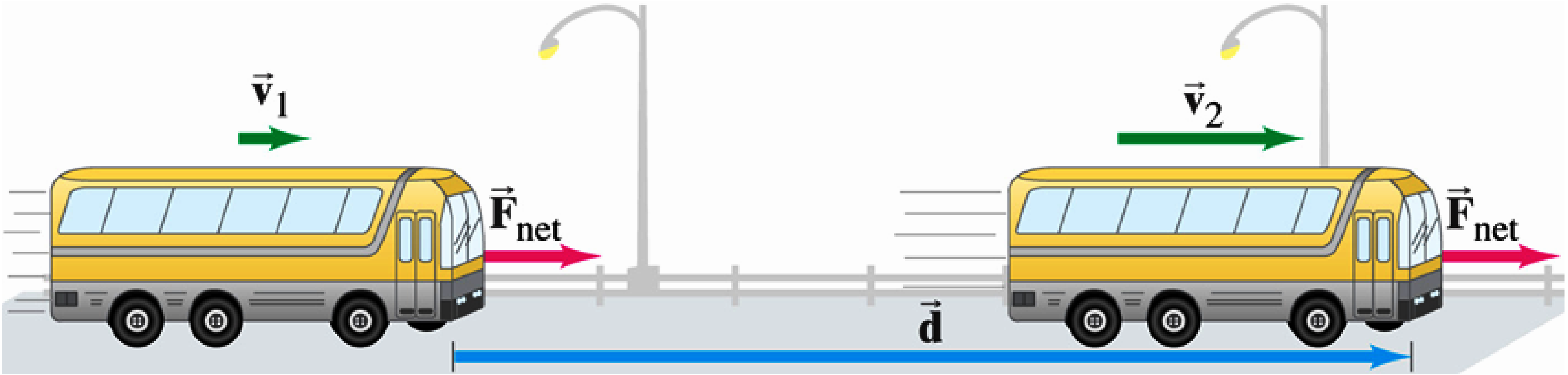
$$W_{net} = KE_2 - KE_1$$

$$W_{net} = (\tfrac{1}{2})m(v_2)^2 - (\tfrac{1}{2})m(v_1)^2$$

W_{net} = **W**ork done by the net (total) force.

ΔKE = Change in Kinetic Energy (KE).

- Consider an object moving in straight line. It starts at speed $\mathbf{v}_1$. Due to the presence of a net force $\mathbf{F}_{net}$, ($\equiv \sum\mathbf{F}$), it accelerates (uniformly) to speed $\mathbf{v}_2$, over a distance $\mathbf{d}$.



Newton's 2nd Law: $\mathbf{F}_{net} = \mathbf{ma}$ (1)

1d motion, constant **a**

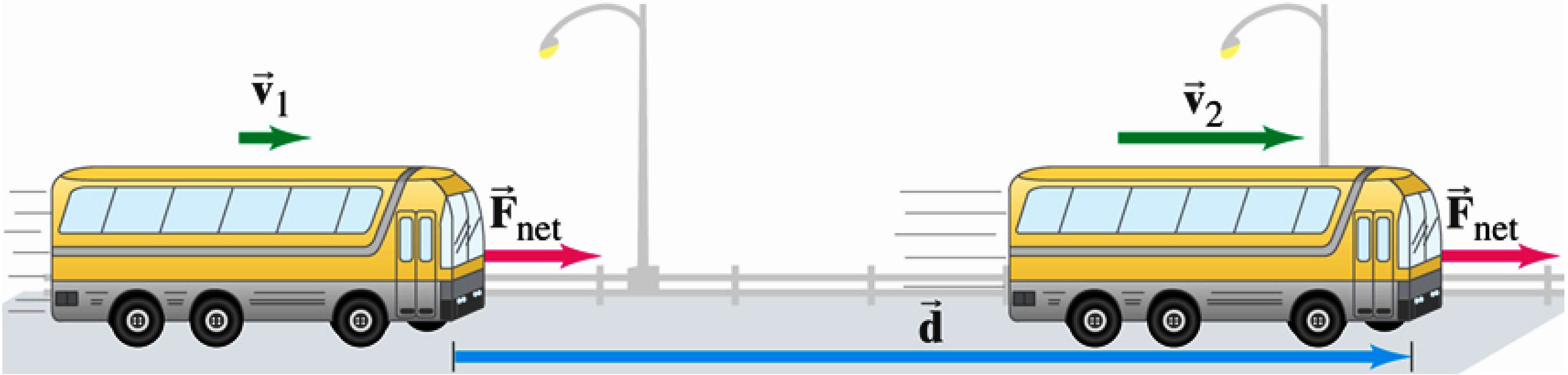
$\Rightarrow \quad (\mathbf{v}_2)^2 = (\mathbf{v}_1)^2 + 2\mathbf{ad}$

$\Rightarrow \quad \mathbf{a} = [(\mathbf{v}_2)^2 - (\mathbf{v}_1)^2]/(2\mathbf{d})$ (2)

Work done: $\mathbf{W}_{net} = \mathbf{F}_{net}\,\mathbf{d}$ (3)

Combine (1), (2), (3):

FIGURE 6–7 A constant net force F_{net} accelerates a bus from speed v_1 to speed v_2 over a distance d. The work done is $W = F_{net}d$.



$$F_{net} = ma \qquad (1)$$

$$a = [(v_2)^2 - (v_1)^2]/(2d) \qquad (2)$$

$$W_{net} = F_{net}\, d \qquad (3)$$

Combine (1), (2) & (3):

$$\Rightarrow W_{net} = m*a*d = m*d\ [(v_2)^2 - (v_1)^2]/(2d)$$

OR

$$W_{net} = (½)m(v_2)^2 - (½)m(v_1)^2$$

- ***Summary:*** The net work done by a constant force in accelerating an object of mass **m** from $\mathbf{v_1}$ to $\mathbf{v_2}$ is:

$$W_{\text{net}} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \equiv \Delta \text{KE}$$

DEFINITION: Kinetic Energy (KE)

(for translational motion; Kinetic = "motion")

$$\text{KE} = \frac{1}{2} m v^2$$

(units are Joules, J)

- We've shown: The ***WORK-ENERGY PRINCIPLE***

$$\mathbf{W_{net} = \Delta KE}$$

(Δ = "change in")

We've shown this for a 1d constant force. However, it is valid in general!

- Net work on an object = Change in KE.

$$W_{net} = \Delta KE \quad (I)$$

$\equiv$ ***The Work-Energy Principle***

Note: W_{net} = work done by the net (total) force.

W_{net} is a scalar & can be positive or negative (because ΔKE can be both + & -). If the net work is positive, the kinetic energy KE increases. If the net work is negative, the kinetic energy KE decreases.

Units are Joules for both work & kinetic energy.

Example: Kinetic energy & work done on a baseball

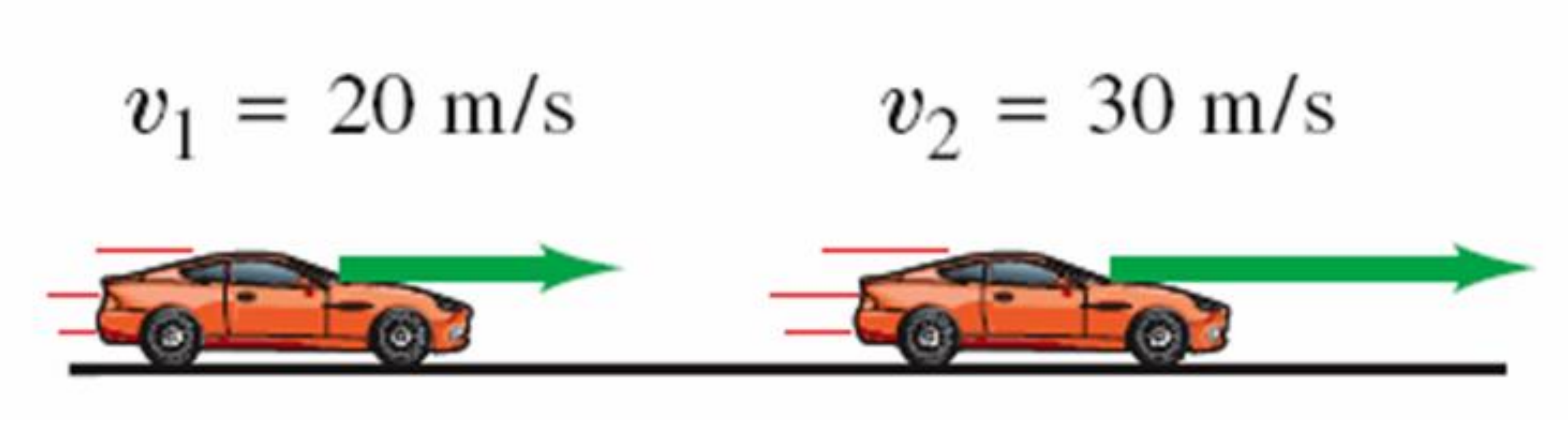
A baseball, mass **m = 145 g** (**0.145 kg**) is thrown so that it acquires a speed **v = 25 m/s**.

a. What is its kinetic energy?

b. What was the net work done on the ball to make it reach this speed, starting from rest?

Ex. 6-4: Work on a car to increase its kinetic energy

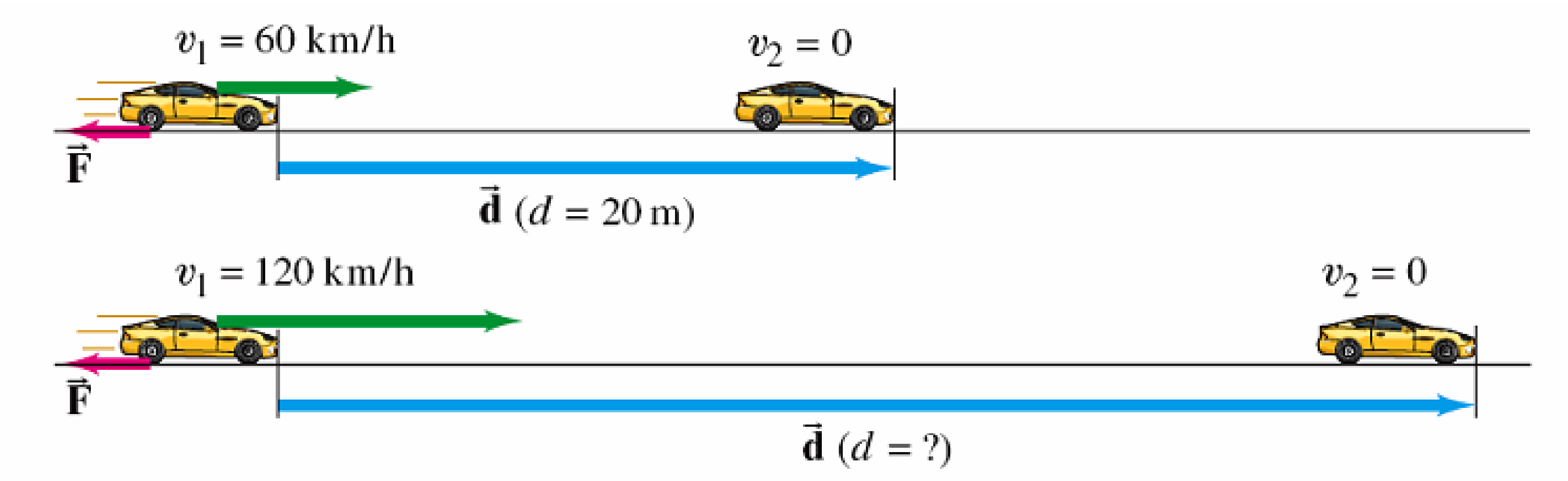
Calculate the net work required to accelerate a car, mass **m = 1000-kg** car from **v_1 = 20 m/s** to **v_2 = 30 m/s**.



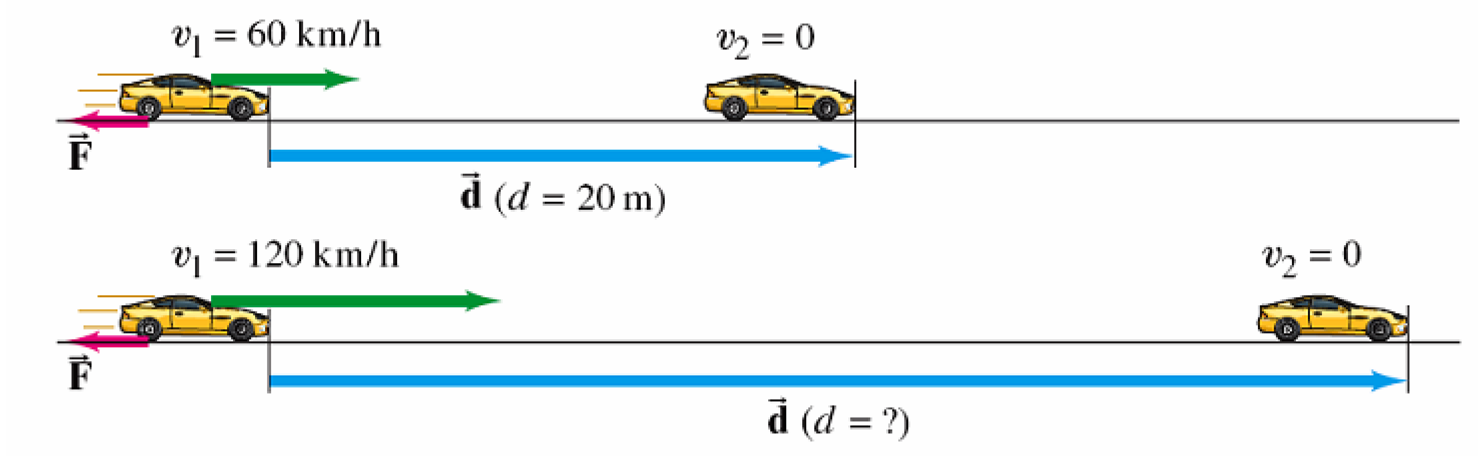
SOLUTION The net work needed is equal to the increase in kinetic energy:

$$
\begin{aligned}
W &= \mathrm{KE}_2 - \mathrm{KE}_1 \\
&= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \\
&= \tfrac{1}{2}(1000\,\mathrm{kg})(30\,\mathrm{m/s})^2 - \tfrac{1}{2}(1000\,\mathrm{kg})(20\,\mathrm{m/s})^2 \\
&= 2.5 \times 10^5\,\mathrm{J}.
\end{aligned}
$$

Conceptual Example 6-5: Work to stop a car



A car traveling at speed $\mathbf{v_1 = 60}$ **km/h** can brake to a stop within a distance **d = 20 m**. If the car is going twice as fast, **120 km/h**, what is its stopping distance? Assume that the maximum braking force is approximately independent of speed.



$W_{net} = Fd\cos(180^\circ) = -Fd$ (from the definition of work)

$W_{net} = \Delta KE = (½)m(v_2)^2 - (½)m(v_1)^2$ (Work-Energy Principle)

but, $(v_2)^2 = 0$ (the car has stopped) so $-Fd = \Delta KE = 0 - (½)m(v_1)^2$

or $d \propto (v_1)^2$

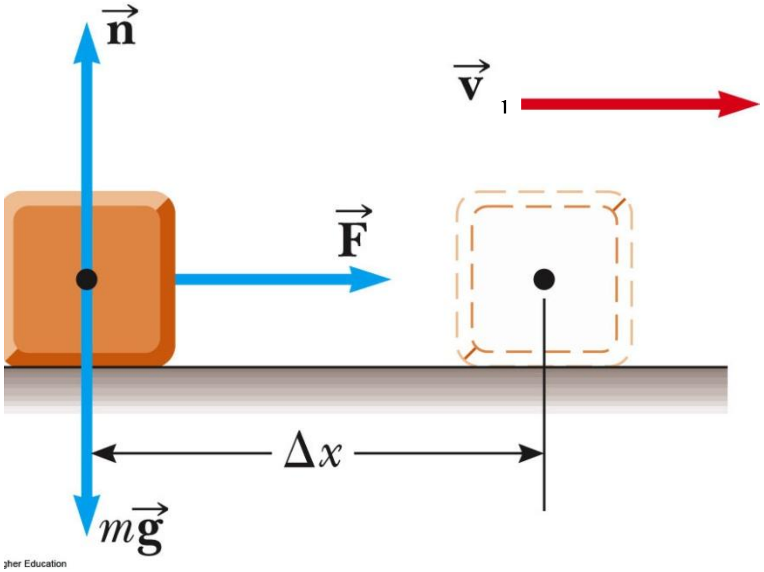
So the stopping distance is proportional to the square of the initial speed!
If the initial speed is doubled, the stopping distance quadruples!

Note: $KE \equiv (½)mv^2 \geq 0$ Must be positive, since **m** & v^2 are always positive .

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

Example

A block, mass **m = 6 kg**, is pulled from rest (**v_0 = 0**) to the right by a constant horizontal force **F = 12 N**. After it has been pulled for **Δx = 3 m**, find it's final speed **v**.



Work-Energy Principle

$$W_{net} = \Delta KE \equiv (½)[m(v)^2 - m(v)^2] \quad (1)$$

If **F = 12 N** is the only horizontal force, we have

$$W_{net} = F\Delta x \quad (2)$$

Combine **(1)** & **(2)**:

$$F\Delta x = (½)[m(v)^2 - 0]$$

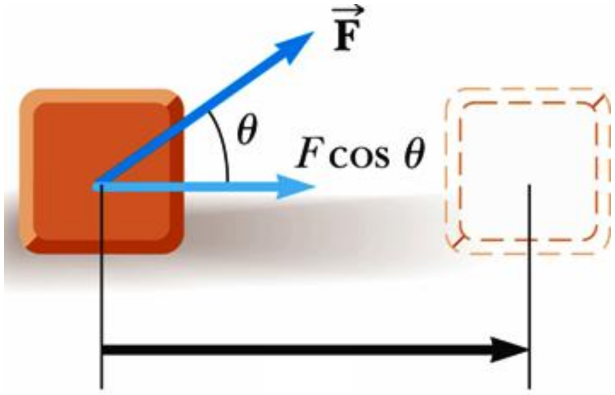
Solve for **v**: $(v)^2 = [2F\Delta x/m]$

$$v = [2F\Delta x/m]^{½} = 3.5\ m/s$$

Summary so Far

- **Work** (constant force):

$$W = F_{\parallel} d = Fd\cos\theta$$



- **Work-Energy Principle:**

$$W_{net} = (½)m(v_2)^2 - (½)m(v_1)^2 \equiv \Delta KE$$

Total work done by *ALL* forces!

- **Kinetic Energy:**

$$KE \equiv (½)mv^2$$

Sect. 6-4: Potential Energy

A mass can have a ***Potential Energy*** due to its environment

Potential Energy **(PE)** $\equiv$

Energy associated with the position or configuration of a mass.

Examples of potential energy:

A wound-up spring

A stretched elastic band

An object at some height above the ground

- ***Potential Energy*** **(PE)** ≡ Energy associated with the position or configuration of a mass.

  Potential work done!

  ***Gravitational Potential Energy*:**

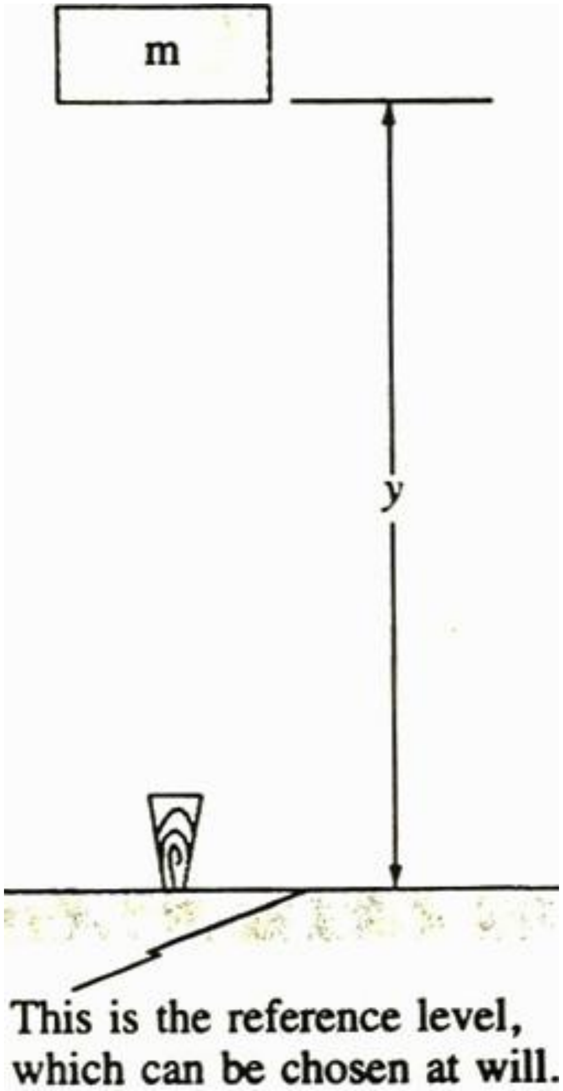
  $$\mathbf{PE_{grav} \equiv mgy}$$

  y = distance above Earth

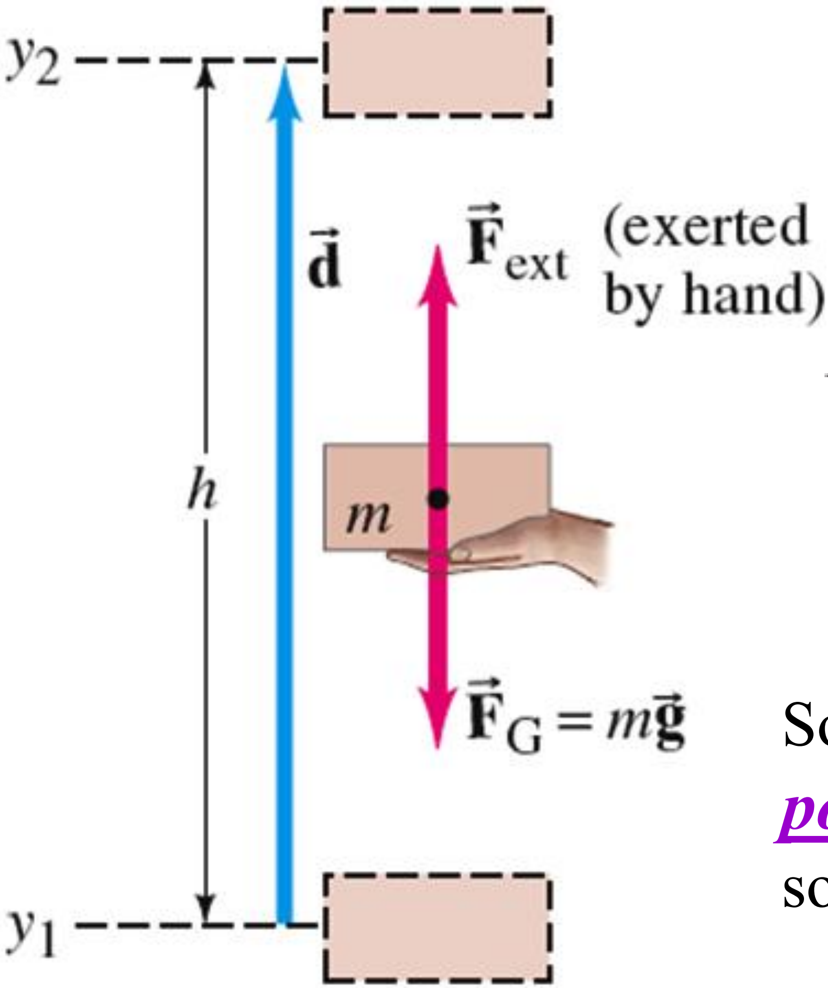
  m has the ***potential*** to do work

  mgy when it falls

  (W = Fy, F = mg)



Gravitational Potential Energy



We know that for constant speed

$$\Sigma F_y = F_{ext} - mg = 0$$

So, in raising a mass **m** to a height **h**, the ***work done by the external force is***

$$W_{ext} = F_{ext} h \cos\theta = mgh\cos 0^\circ$$

$$= mgh = mg(y_2 - y_1)$$

So we ***define the gravitational potential energy*** at a height **y** above some reference point (**y_1**) as

$$(\mathrm{PE})_{grav} = mgy$$

- Consider a problem in which the height of a mass above the Earth changes from $\mathbf{y_1}$ to $\mathbf{y_2}$:

- ***The Change in Gravitational PE*** is:

$$\mathbf{\Delta(PE)_{grav} = mg(y_2 - y_1)}$$

- Work done on the mass: $\mathbf{W = \Delta(PE)_{grav}}$

$\mathbf{y}$ = distance above Earth

Where we choose $\mathbf{y = 0}$ is ***arbitrary***, since we take the difference in 2 $\mathbf{y}$'s in $\mathbf{\Delta(PE)_{grav}}$

Of course, this **potential energy can be converted to kinetic energy if the object is dropped**.

Potential energy is a property of a system as a whole, not just of the object (because it depends on external forces).

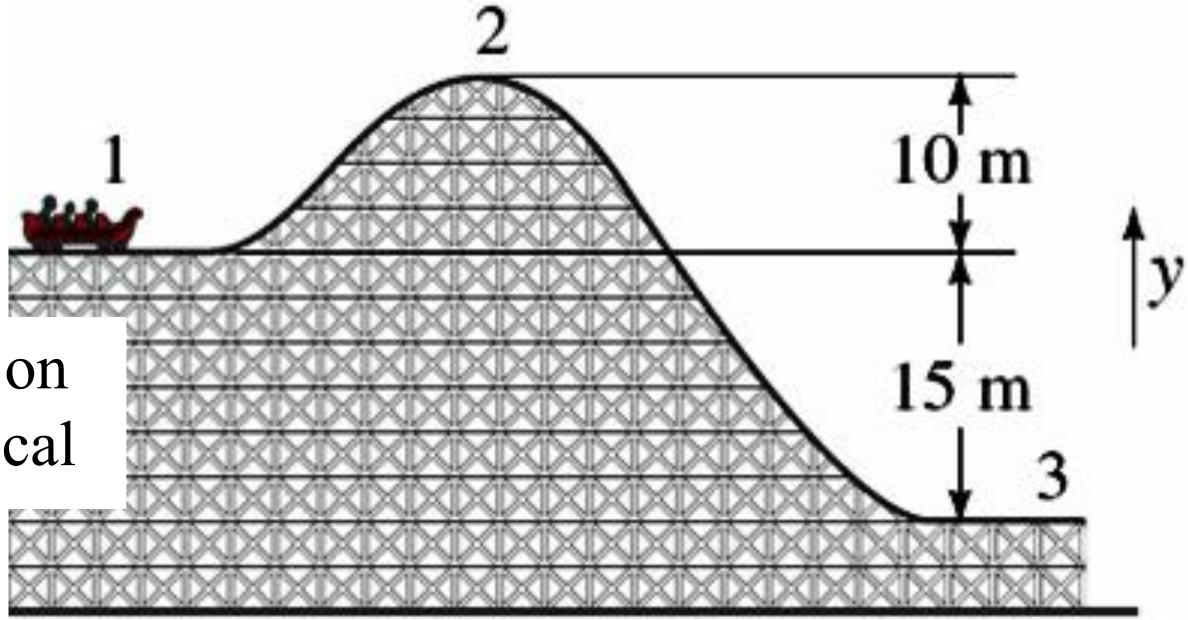
If $\mathbf{PE_{grav} = mgy}$, from where do we measure $\mathbf{y}$?

It turns out not to matter,

as long as we are consistent about where we choose $\mathbf{y = 0}$. Because only **changes** in potential energy can be measured.

Example 6-6: Potential energy changes for a roller coaster

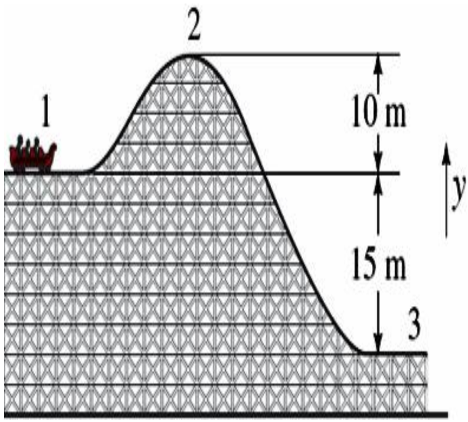
A roller-coaster car, mass **m = 1000 kg**, moves from point **1** to point **2** & then to point **3**.



ΔPE depends only on ***differences*** in vertical height.

a. Calculate the gravitational potential energy at points **2** & **3** relative to point **1**. (That is, take **y = 0** at point **1**.) **b.** Calculate the ***change*** in potential energy when the car goes from point **2** to point **3**.
c. Repeat parts **a.** & **b.**, but take the reference point (**y = 0**) at point **3**.

Example 6-6: Potential energy changes for a roller coaster



SOLUTION (*a*) We measure heights from point 1 $(y_1 = 0)$, which means initially that the gravitational potential energy is zero. At point 2, where $y_2 = 10\text{ m}$,

$$\text{PE}_2 = mgy_2 = (1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 9.8 \times 10^4\text{ J}.$$

At point 3, $y_3 = -15\text{ m}$, since point 3 is below point 1. Therefore,

$$\text{PE}_3 = mgy_3 = (1000\text{ kg})(9.8\text{ m/s}^2)(-15\text{ m}) = -1.5 \times 10^5\text{ J}.$$

(*b*) In going from point 2 to point 3, the potential energy change $(\text{PE}_{\text{final}} - \text{PE}_{\text{initial}})$ is

$$\text{PE}_3 - \text{PE}_2 = (-1.5 \times 10^5\text{ J}) - (9.8 \times 10^4\text{ J}) = -2.5 \times 10^5\text{ J}.$$

The gravitational potential energy decreases by 2.5×10^5 J.

(*c*) Now we set $y_3 = 0$. Then $y_1 = +15\text{ m}$ at point 1, so the potential energy initially is

$$\text{PE}_1 = (1000\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 1.5 \times 10^5\text{ J}.$$

At point 2, $y_2 = 25\text{ m}$, so the potential energy is

$$\text{PE}_2 = 2.5 \times 10^5\text{ J}.$$

At point 3, $y_3 = 0$, so the potential energy is zero. The change in potential energy going from point 2 to point 3 is

$$\text{PE}_3 - \text{PE}_2 = 0 - 2.5 \times 10^5\text{ J} = -2.5 \times 10^5\text{ J},$$

which is the same as in part (*b*).

Sect. 6-5: Conservative Forces

- ***Conservative Force*** ≡ The work done by that force depends only on initial & final conditions & not on path taken between the initial & final positions of the mass.

⇒ ***A (PE) CAN be defined for conservative forces***

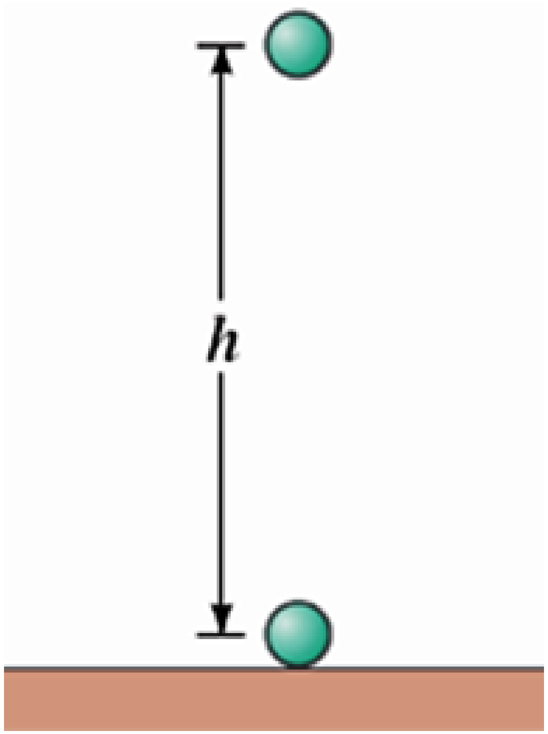
- ***Non-Conservative Force*** ≡ The work done by that force depends on the path taken between the initial & final positions of the mass.

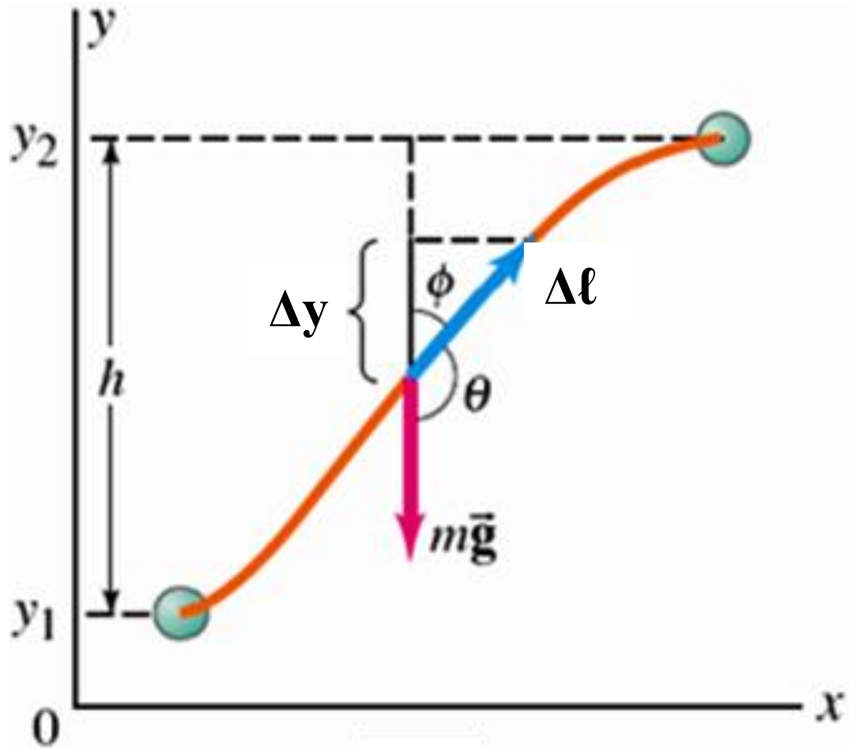
⇒ ***A (PE) CANNOT be defined for non-conservative forces***

- The most common example of a non-conservative force is

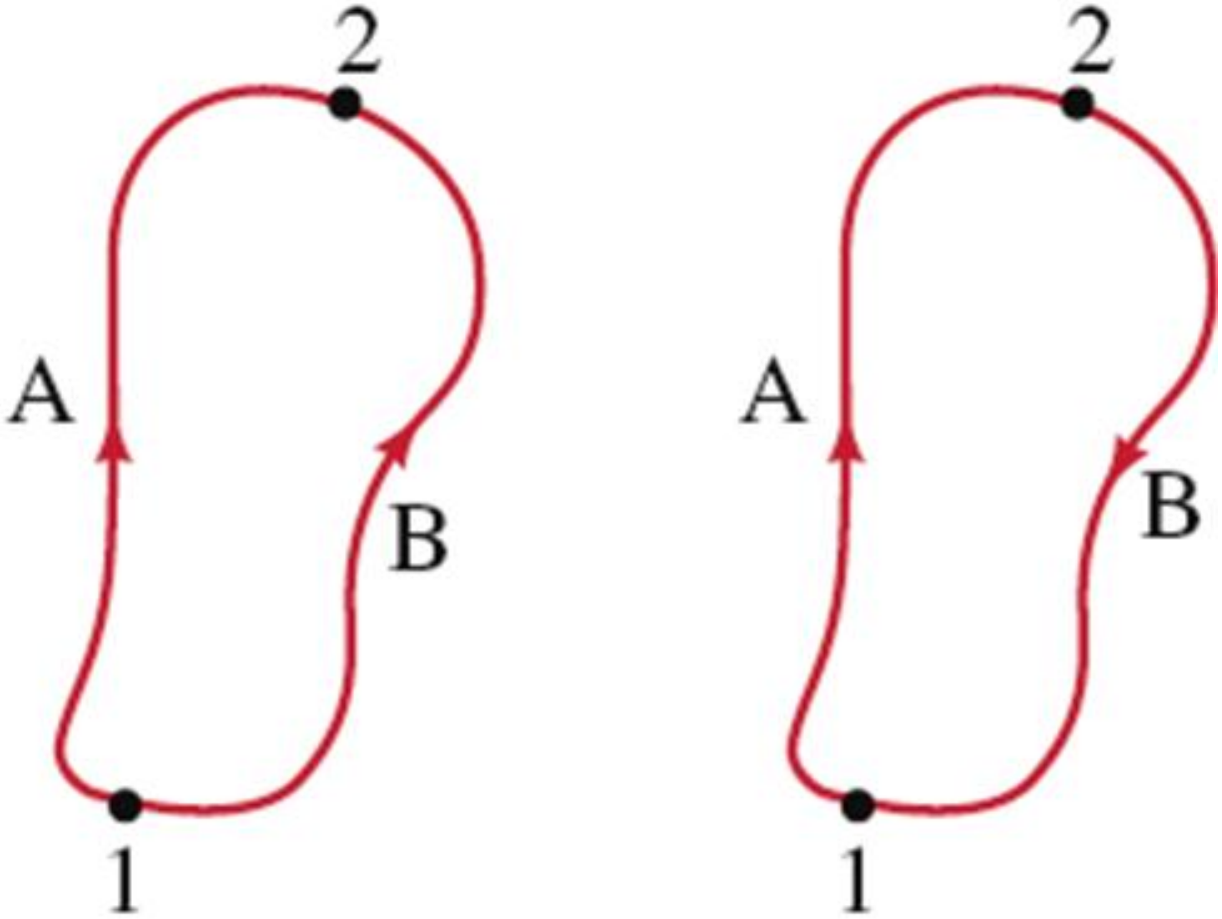
FRICTION

Definition: A force is **conservative** if & only if ***the work done by that force on an object moving from one point to another depends ONLY on the initial & final positions of the object, & is independent of the particular path taken.*** **Example:** gravity.





Conservative Force: Another definition: ***A force is conservative if the net work done by the force on an object moving around any closed path is zero.***



Potential Energy:

Can only be defined for Conservative Forces!

In other words,

If a force is **Conservative**, a (PE) ***CAN*** be defined.

But,

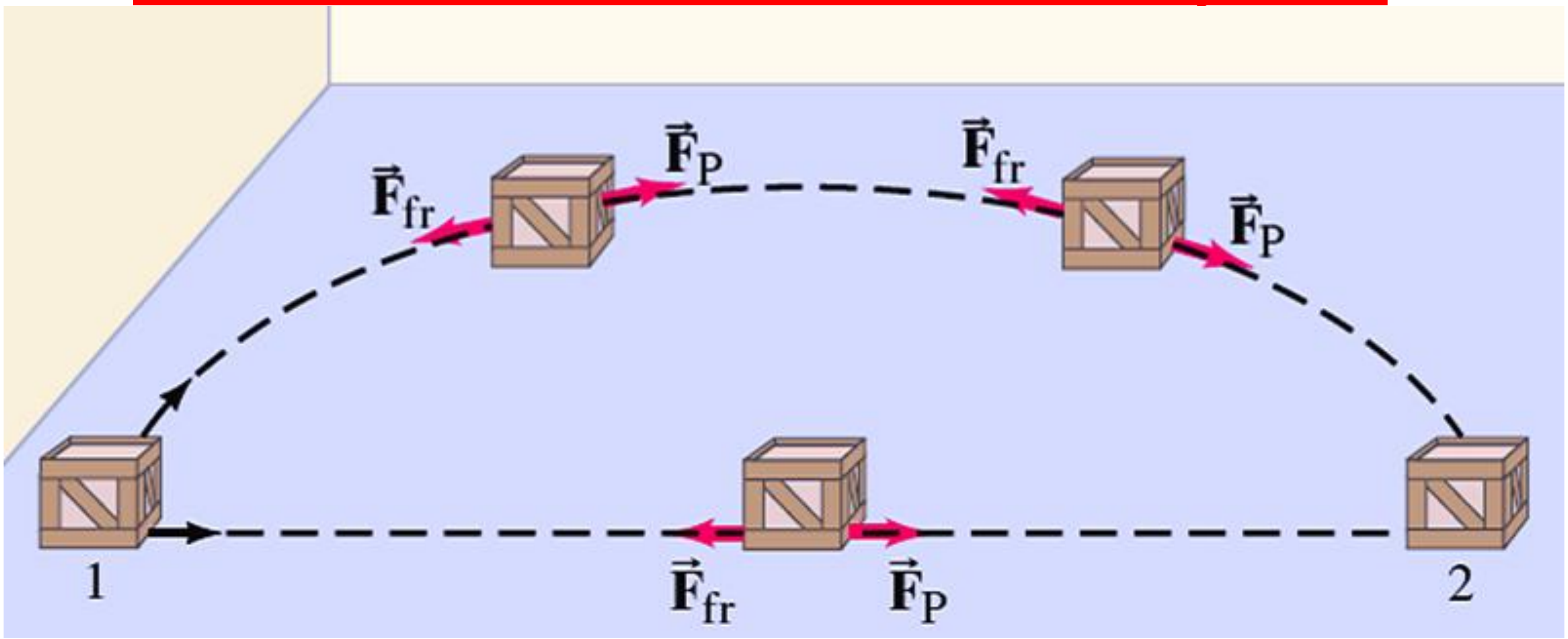
If a force is **Non-Conservative**, a (PE) ***CANNOT*** be defined!!

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by a person

If **friction** is present, the work done depends not only on the starting & ending points, but also on the path taken.

Friction is a non-conservative force!



Friction is non-conservative!!!

The work done depends on the path!

- If several forces act, (conservative & non-conservative), the total work done is: $\mathbf{W_{net} = W_C + W_{NC}}$

  $\mathbf{W_C}$ ≡ work done by conservative forces

  $\mathbf{W_{NC}}$ ≡ work done by non-conservative forces
- ***The work energy principle*** still holds:

$$\mathbf{W_{net} = \Delta KE}$$

- For conservative forces (by the **definition of PE**):

$$\mathbf{W_C = -\Delta PE}$$

$$\Rightarrow \qquad \mathbf{\Delta KE = -\Delta PE + W_{NC}}$$

or: $$\mathbf{W_{NC} = \Delta KE + \Delta PE}$$

$\Rightarrow$ In general, $\mathbf{W_{NC} = \Delta KE + \Delta PE}$

The total work done by all non-conservative forces

$\equiv$

The total change in KE + The total change in PE

Sect. 6-6: Mechanical Energy & its Conservation

***GENERALLY**: In any process, total energy is neither created nor destroyed.*

- Energy can be transformed from one form to another & from one object to another, but the

Total Amount Remains Constant.

≡ Law of Conservation of Total Energy

- In general, for mechanical systems, we just found:

$$\mathbf{W_{NC} = \Delta KE + \Delta PE}$$

For the ***Very Special Case*** of ***Conservative Forces Only***

$$\Rightarrow \qquad \mathbf{W_{NC} = 0}$$

$$\Rightarrow \qquad \mathbf{\Delta KE + \Delta PE = 0}$$

≡ ***The Principle of Conservation of Mechanical Energy***

- **Please Note!!** This is ***NOT*** (quite) the same as the Law of Conservation of Total Energy! It is a ***very special case*** of this law (where all forces are conservative)

- So, for conservative forces ***ONLY!*** In any process

$$\Delta KE + \Delta PE = 0$$

Conservation of Mechanical Energy

- It is convenient to ***Define*** the Mechanical Energy: **E ≡ KE + PE**

⇒ In any process (conservative forces!): $\Delta E = 0 = \Delta KE + \Delta PE$

Or, **E = KE + PE = Constant**

≡ **Conservation of Mechanical Energy**

In any process (conservative forces!), the sum of the KE & the PE is unchanged:
That is, the mechanical energy may change from PE to KE or from KE to PE, but

Their Sum Remains Constant.

Principle of Conservation of Mechanical Energy

If only conservative forces are doing work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

- ***Conservation of Mechanical Energy:***

$$\Rightarrow \quad \Delta KE + \Delta PE = 0$$

or

$$E = KE + PE = \text{Constant}$$

For conservative forces ***ONLY*** (gravity, spring, etc.)

- Suppose that, initially: $E = KE_1 + PE_1$, & finally: $E = KE_2 + PE_2$. But, $E = \text{Constant}$, so

$$\Rightarrow \quad KE_1 + PE_1 = KE_2 + PE_2$$

A *very powerful* method of calculation!!

- ***Conservation of Mechanical Energy***

$\Rightarrow$ $\Delta KE + \Delta PE = 0$ or

$$E = KE + PE = \text{Constant}$$

For gravitational PE: $(PE)_{grav} = mgy$

$$E = KE_1 + PE_1 = KE_2 + PE_2$$

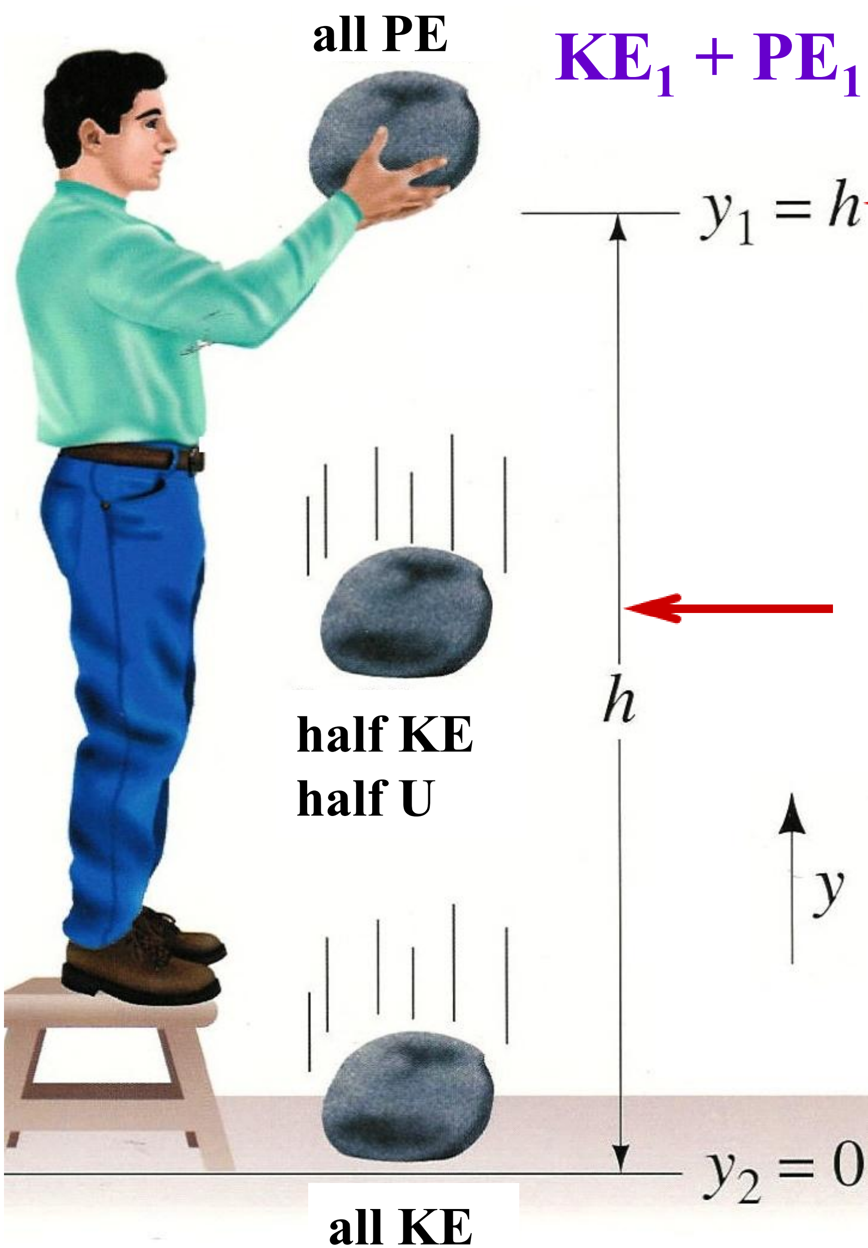
$$\Rightarrow (\tfrac{1}{2})m(v_1)^2 + mgy_1 = (\tfrac{1}{2})m(v_2)^2 + mgy_2$$

y_1 = Initial height, v_1 = Initial velocity

y_2 = Final height, v_2 = Final velocity

Section 6.7
: Problem Solving Using Conservation of Mechanical Energy

$$KE_1 + PE_1 = KE_2 + PE_2 = KE_3 + PE_3$$



$PE_1 = mgh$, $KE_1 = 0$

FIGURE 6–11 The stone's PE changes to KE as it falls.

but their sum remains constant!

$$KE_3 + PE_3 = KE_2 + PE_2$$
$$= KE_1 + PE_1$$

$$KE_1 + PE_1 = KE_2 + PE_2$$
$$0 + mgh = (½)mv^2 + 0$$
$$v^2 = 2gh$$

$PE_2 = 0$

$KE_2 = (½)mv^2$

Energy “buckets” are not real!!

Example 6-7: Falling Rock

- Speed at **y = 1.0 m**?

Mechanical Energy Conservation!

$$(\tfrac{1}{2})m(v_1)^2 + mgy_1 =$$
$$(\tfrac{1}{2})m(v_2)^2 + mgy_2 =$$
$$(\tfrac{1}{2})m(v_3)^2 + mgy_3$$ (Mass cancels!)

$y_1 = 3.0$ m, $v_1 = 0$, $y_2 = 1.0$ m, $v_2 = ?$

Result: $v_2 = 6.3$ m/s

NOTE!! ***Always*** **use**

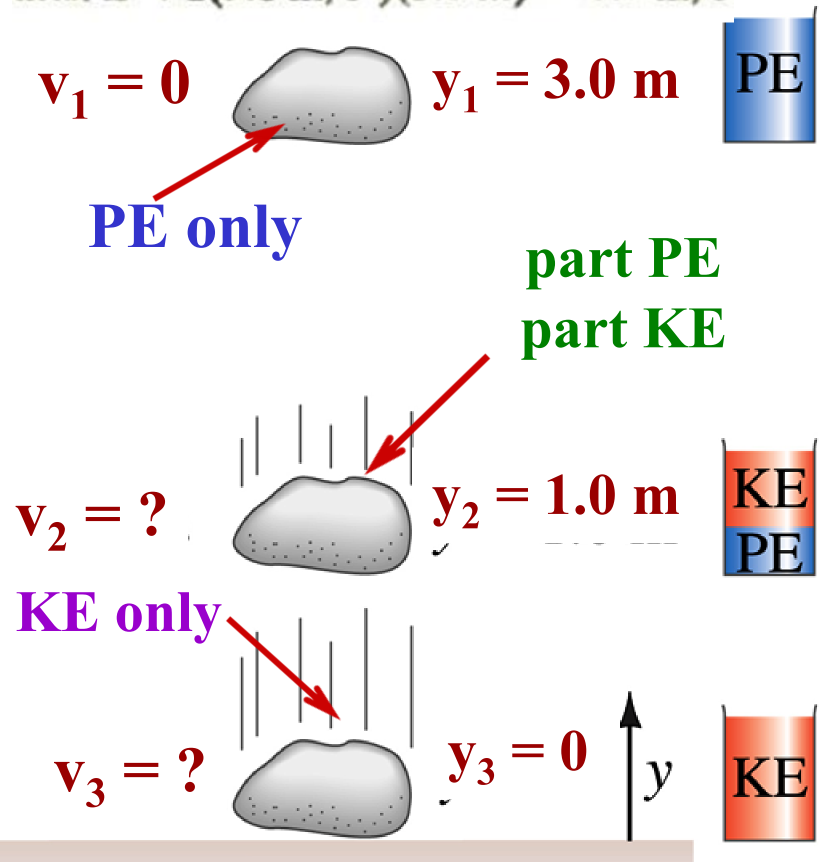
$KE_1 + PE_1 = KE_2 + PE_2 = KE_3 + PE_3$

NEVER $KE_3 = PE_3$!!!!

A very common error!

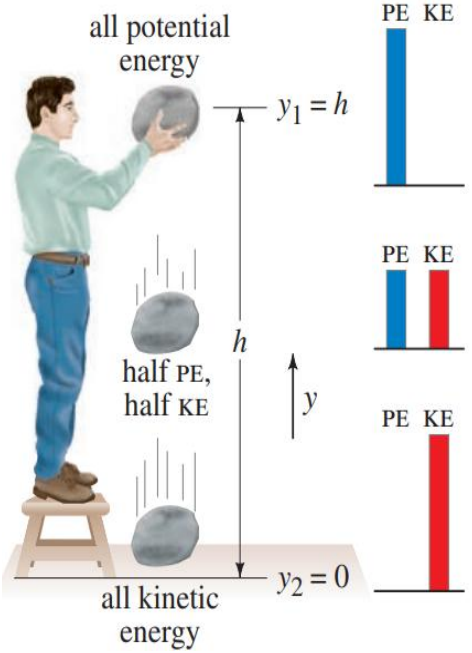
WHY???? *In general,* $KE_3 \neq PE_3$!!!

FIGURE 6–18 Energy buckets (for Example 6–8). Kinetic energy is red and potential energy is blue. The total (KE + PE) is the same for the three points shown. Note that the speed at $y = 0$, just before the rock hits, is $\sqrt{2(9.8\ \text{m/s}^2)(3.0\ \text{m})} = 7.7$ m/s.



Example 6-7: Falling Rock

EXAMPLE 6–7 **Falling rock.** If the initial height of the rock in Fig. 6–17 is $y_1 = h = 3.0\,\text{m}$, calculate the rock's velocity when it has fallen to 1.0 m above the ground.



We choose the ground as our reference level (y = 0)

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\,\text{m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\,\text{m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 .$$

The m's cancel out and $v_1 = 0$, so

$$gy_1 = \tfrac{1}{2}v_2^2 + gy_2 .$$

Solving for v_2 we find

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.8\,\text{m/s}^2)[(3.0\,\text{m}) - (1.0\,\text{m})]} = 6.3\,\text{m/s}.$$

The rock's velocity 1.0 m above the ground is 6.3 m/s downward.

Example 6-8: Roller Coaster

- ***Mechanical energy conservation!*** **(Frictionless!)**

$\Rightarrow (\frac{1}{2})m(v_1)^2 + mgy_1 = (\frac{1}{2})m(v_2)^2 + mgy_2$ (Mass cancels!)

Only height differences matter! **Horizontal distance doesn't matter!**

- Speed at the bottom?

  $y_1 = 40$ m, $v_1 = 0$

  $y_2 = 0$ m, $v_2 = ?$

  Find: $v_2 = 28$ m/s

- What is y when

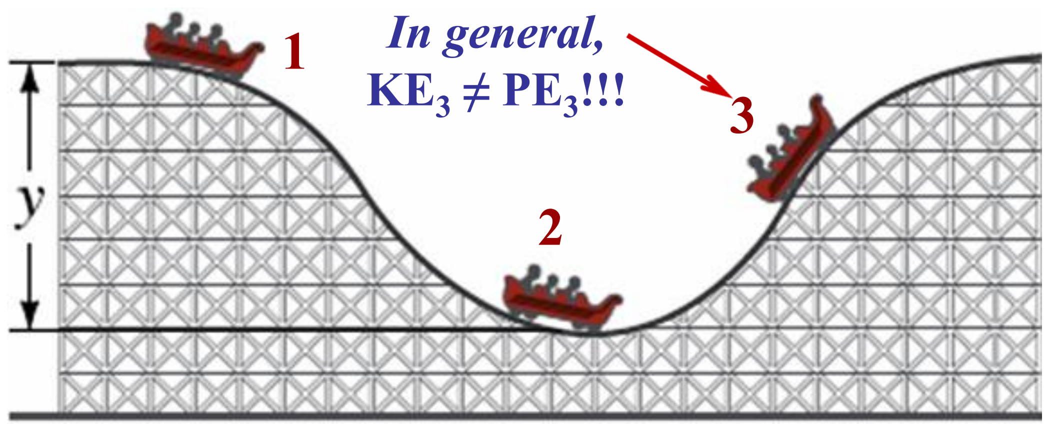
  $v_3 = 14$ m/s?

Use: $(\frac{1}{2})m(v_2)^2 + 0$

$= (\frac{1}{2})m(v_3)^2 + mgy_3$

Find: $y_3 = 30$ m

Height of hill = **40 m**. Car starts from rest at top. Calculate: **a.** Speed of the car at bottom of hill. **b.** Height at which it will have half this speed. Take **y = 0** at bottom of hill.

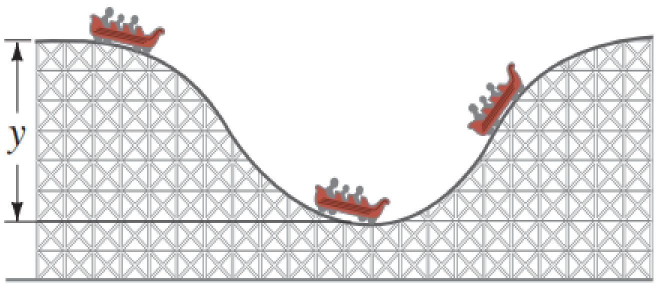


A very common error! ***WHY????***

NOTE!! *Always* use $KE_1 + PE_1 = KE_2 + PE_2 = KE_3 + PE_3$ *Never* $KE_3 = PE_3$!

Example 6-8: Roller Coaster

Assuming the height of the hill in Fig. 6–18 is 40 m, and the roller-coaster car starts from rest at the top, calculate (a) the speed of the roller-coaster car at the bottom of the hill, and (b) at what height it will have half this speed. Take y = 0 at the bottom of the hill.



APPROACH We use conservation of mechanical energy. We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\,\mathrm{m})$. In part (*a*), point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. In part (*b*) we let y_2 be the unknown.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$, which gives

$$mgy_1 = \tfrac{1}{2}mv_2^2$$

or

$$\begin{aligned} v_2 &= \sqrt{2gy_1} \\ &= \sqrt{2(9.8\,\mathrm{m/s^2})(40\,\mathrm{m})} = 28\,\mathrm{m/s}. \end{aligned}$$

(*b*) Now y_2 will be an unknown. We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = \tfrac{1}{2}(28\,\mathrm{m/s}) = 14\,\mathrm{m/s}$ and $v_1 = 0$. Solving for the unknown y_2 gives

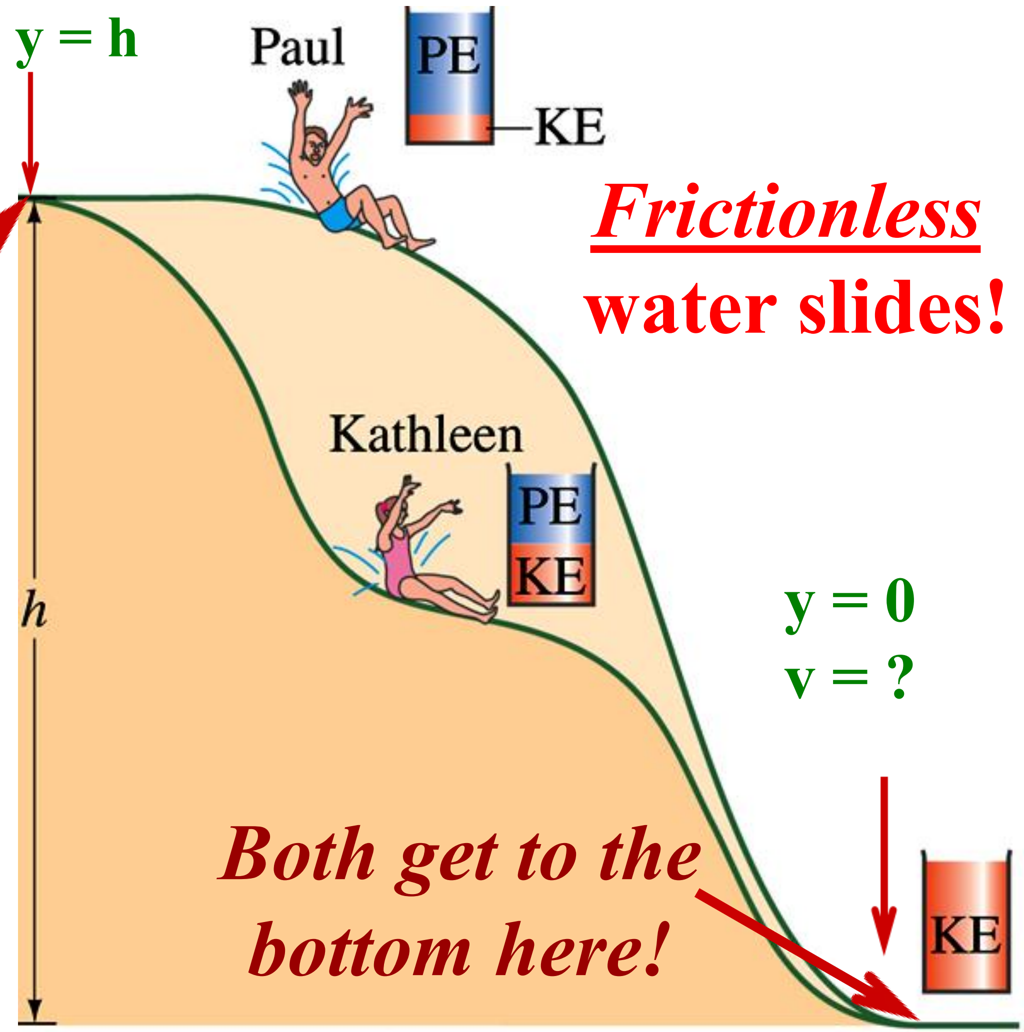
$$y_2 = y_1 - \frac{v_2^2}{2g} = 40\,\mathrm{m} - \frac{(14\,\mathrm{m/s})^2}{2(9.8\,\mathrm{m/s^2})} = 30\,\mathrm{m}.$$

Conceptual Example 6-9: Speeds on 2 Water Slides

- Who is traveling faster at the bottom?
- Who reaches the bottom first?

Both start here!

- Demonstration!



***Frictionless* water slides!**

Both get to the bottom here!

Conceptual Example 6-9: Speeds on 2 Water Slides

Speeds on two water slides. Two water slides at a pool are shaped differently, but start at the same height h (Fig. 6–19). Two riders start from rest at the same time on different slides. (a) Which rider, Paul or Corinne, is traveling faster at the bottom? (b) Which rider makes it to the bottom first? Ignore friction and assume both slides have the same path length.

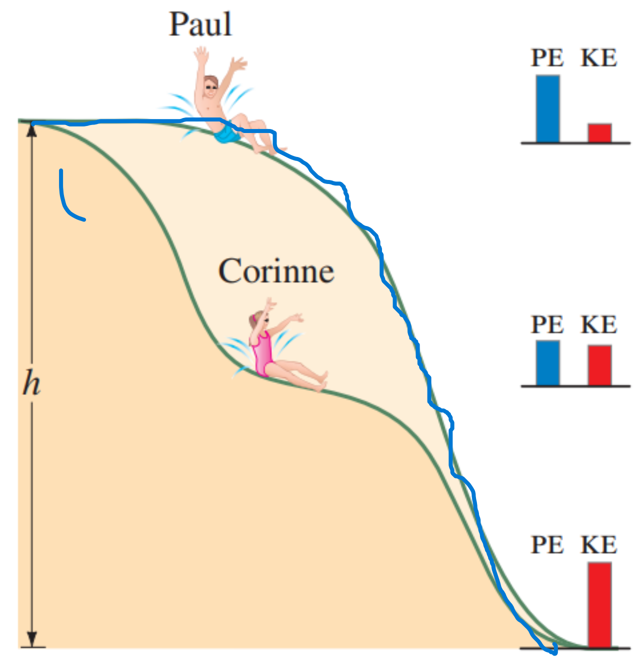


FIGURE 6–19 Example 6–9.

RESPONSE (*a*) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\frac{1}{2}mv^2 = mgh$. The mass cancels and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed. (*b*) Note that Corinne is consistently at a lower elevation than Paul at any instant, until the end. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, and because the distance is the same, Corinne gets to the bottom first.

6–8 Other Forms of Energy and Energy Transformations; The Law of Conservation of Energy

LAW OF CONSERVATION OF ENERGY

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

Sect. 6-9: Problems with Friction

- We had, in general:

$$\mathbf{W_{NC} = \Delta KE + \Delta PE}$$

$\mathbf{W_{NC}}$ = Work done by all non-conservative forces

$\mathbf{\Delta KE}$ = Change in KE

$\mathbf{\Delta PE}$ = Change in PE (conservative forces only)

- ***Friction is a non-conservative force!*** So, if friction is present, we have ($\mathbf{W_{NC} \rightarrow W_{fr}}$)

$\mathbf{W_{fr}}$ = Work done by friction

Moving through a distance **d**, friction force $\mathbf{F_{fr}}$ does work

$$\mathbf{W_{fr} = - \ F_{fr}d}$$

When friction is present, we have:

$$\mathbf{W_{fr} = -F_{fr}d = \Delta KE + \Delta PE = KE_2 - KE_1 + PE_2 - PE_1}$$

- Also now, **KE + PE ≠ Constant!**
- Instead, $\mathbf{KE_1 + PE_1 + W_{fr} = KE_2 + PE_2}$

  or: $\mathbf{KE_1 + PE_1 - F_{fr}d = KE_2 + PE_2}$

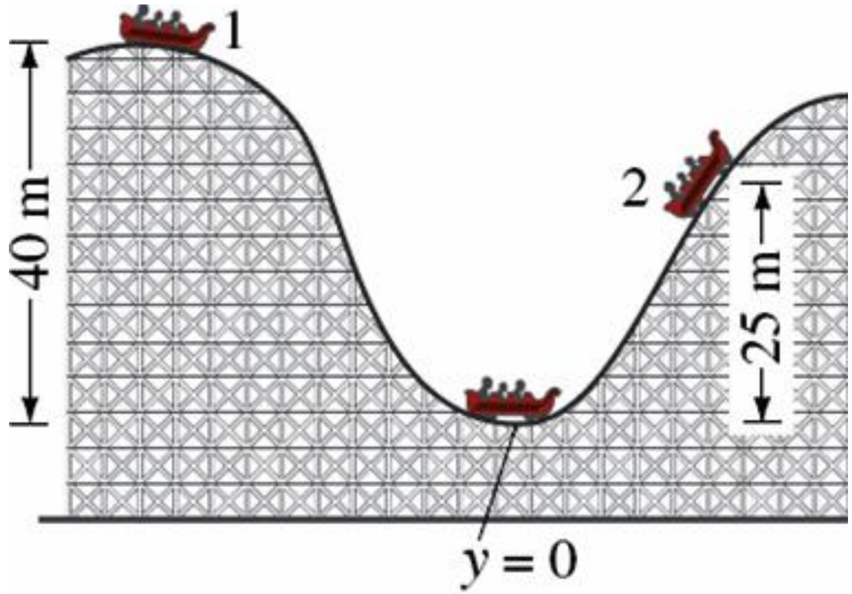
- For ***gravitational*** **PE**:

$$\mathbf{(½)m(v_1)^2 + mgy_1 = (½)m(v_2)^2 + mgy_2 + F_{fr}d}$$

Example 6-12: Roller Coaster with Friction

A roller-coaster car, mass **m = 1000 kg**, reaches a vertical height of only **y = 25 m** on the second hill before coming to a momentary stop. It travels a total distance **d = 400 m**.

Calculate the work done by friction (the thermal energy produced) & calculate the average friction force on the car.



$$\mathbf{m = 1000\ kg,\ d = 400\ m,\ y_1 = 40\ m,\ y_2 = 25\ m,\ v_1 = v_2 = 0,\ F_{fr}} = ?$$

$$\mathbf{(½)m(v_1)^2 + mgy_1 = (½)m(v_2)^2 + mgy_2 + F_{fr}d}$$

the energy dissipated to thermal energy

$$F_{fr}\,d \;=\; mg\,\Delta h \;=\; (1000\,\text{kg})(9.8\,\text{m/s}^2)(40\,\text{m} - 25\,\text{m}) \;=\; 147{,}000\,\text{J}.$$

The friction force, which acts over a distance of 400 m, averages out to be

$$F_{fr} = (1.47 \times 10^5\,\text{J})/400\,\text{m} = 370\,\text{N}.$$

Sect. 6-10: Power

Power ≡ ***Rate at which work is done or rate at which energy is transformed***:

- **Average Power**:

  P = (Work)/(Time) = (Energy)/(time) $\overline{P} = \frac{W}{t}.$

- **Instantaneous power:**

  SI units: Joule/Second = Watt (W) **1 W = 1J/s**

  British units: Horsepower (hp). 1hp = 746 W

A side note:

"Kilowatt-Hours" (from your power bill). **Energy!**

1 KWh = (10^3 Watt) × (3600 s) = 3.6 × 10^6 W s = 3.6 × 10^6 J

Example 6-13: Stair Climbing Power

A **60-kg** jogger runs up a long flight of stairs in **4.0 s**. The vertical height of the stairs is **4.5 m**.

a. Estimate the jogger's power output in watts and horsepower.

b. How much energy did this require?

SOLUTION (*a*) The average power output was

$$\overline{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(60\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(4.5\,\mathrm{m})}{4.0\,\mathrm{s}} = 660\,\mathrm{W}.$$

Since there are 746 W in 1 hp, the jogger is doing work at a rate of just under 1 hp.

(*b*) The energy required is $E = \overline{P}t = (660\,\mathrm{J/s})(4.0\,\mathrm{s}) = 2600\,\mathrm{J}$. This result equals $W = mgy$.

- ***<u>Average Power</u>*** $\overline{P} = \frac{W}{t}.$

- Its often convenient to write power in terms of force & speed. For example, for a force **F** & displacement **d** in the same direction, we know that the work done is:

$$\mathbf{W = F\ d} \qquad \text{So}$$

$$\Rightarrow \overline{P} = \mathbf{F\ (d/t) = F\ v} = \text{average power}$$

$$\mathbf{v} \equiv \text{Average speed of the object}$$

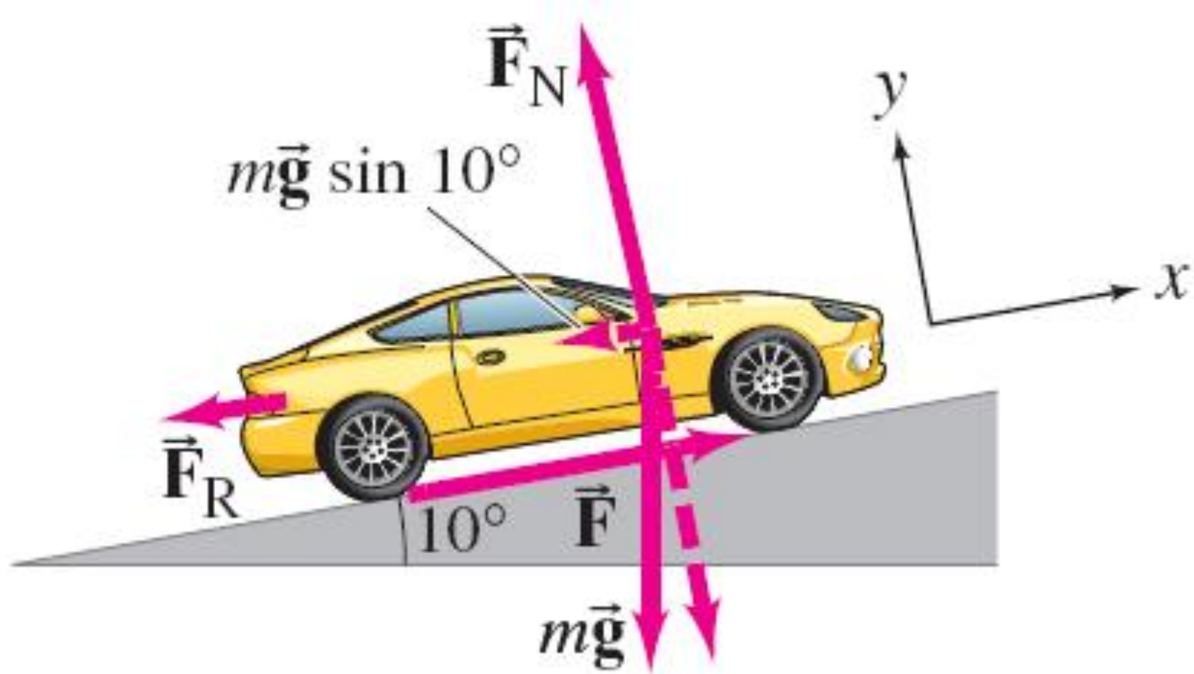
Example 6-14: Power needs of a car

Calculate the power required for a **1400-kg** car to do the following:

a. Climb a **10°** hill (steep!) at a steady **80 km/h**

b. Accelerate on a level road from **90** to **110 km/h** in **6.0 s**

Assume that the average retarding force on the car is $\mathbf{F_R}$ **= 700 N**.



Example 6-14: Power needs of a car

a. $\sum F_x = 0$, $F - F_R - mg\sin\theta = 0$, $F = F_R + mg\sin\theta$

$$F = 700\,\text{N} + mg\sin 10^\circ$$
$$= 700\,\text{N} + (1400\,\text{kg})(9.80\,\text{m/s}^2)(0.174) = 3100\,\text{N}.$$

Since $\bar{v} = 80\,\text{km/h} = 22\,\text{m/s}^\dagger$ and is parallel to $\vec{\mathbf{F}}$, then

$$\overline{P} = F\bar{v} = (3100\,\text{N})(22\,\text{m/s}) = 6.8 \times 10^4\,\text{W} = 68\,\text{kW} = 91\,\text{hp}.$$

b. $$\bar{a}_x = \frac{(30.6\,\text{m/s} - 25.0\,\text{m/s})}{6.0\,\text{s}} = 0.93\,\text{m/s}^2.$$ $$ma_x = \Sigma F_x = F - F_R.$$

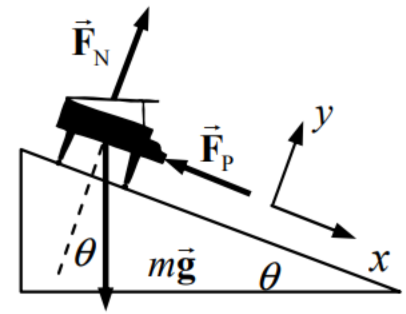
$$F = ma_x + F_R$$
$$= (1400\,\text{kg})(0.93\,\text{m/s}^2) + 700\,\text{N} = 1300\,\text{N} + 700\,\text{N} = 2000\,\text{N}.$$

Since $\overline{P} = F\bar{v}$, the required power increases with speed and the motor must be able to provide a maximum power output in this case of

$$\overline{P} = (2000\,\text{N})(30.6\,\text{m/s}) = 6.1 \times 10^4\,\text{W} = 61\,\text{kW} = 82\,\text{hp}.$$

Problem 10

(II) A 380-kg piano slides 2.9 m down a 25° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (Fig. 6–36). Determine: (*a*) the force exerted by the man, (*b*) the work done on the piano by the man, (*c*) the work done on the piano by the force of gravity, and (*d*) the net work done on the piano. Ignore friction.

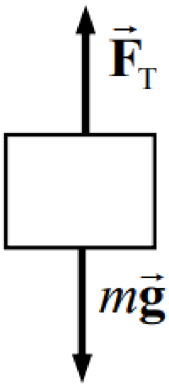


Problem 18

How much work must be done to stop a 925-kg car traveling at 95 km/h?

Problem 23

A 265-kg load is lifted 18.0 m vertically with an acceleration a = 0.160 g by a single cable. Determine (a) the tension in the cable; (b) the net work done on the load; (c) the work done by the cable on the load; (d) the work done by gravity on the load; (e) the final speed of the load assuming it started from rest.



Problem 28

A 1.60-m-tall person lifts a 1.65-kg book off the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to (a) the ground, and (b) the top of the person's head? (c) How is the work done by the person related to the answers in parts (a) and (b)?

Problem 36

A roller-coaster car shown in Fig. 6–41 is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3, and 4

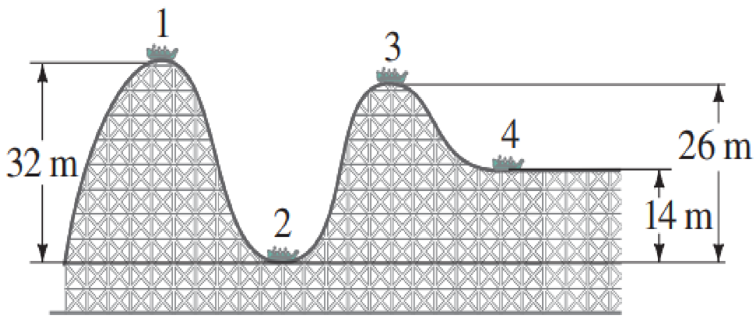


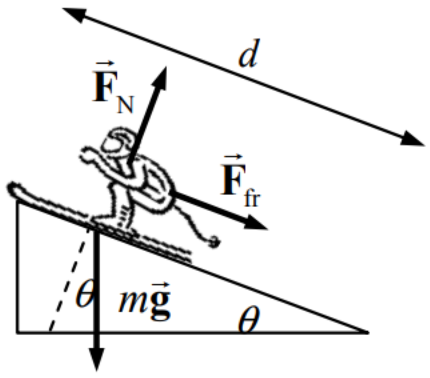
FIGURE 6–41 Problems 36.

Problem 41

A 16.0-kg child descends a slide 2.20 m high and, starting from rest, reaches the bottom with a speed of 1.25 m/s. How much thermal energy due to friction was generated in this process?

Problem 44

A skier traveling 11.0 m/s reaches the foot of a steady upward 19° incline and glides 15 m up along this slope before coming to rest. What was the average coefficient of friction?



Problem 55

How much work can a 2.0-hp motor do in 1.0 h?

Power

1500 (60×60)s

W = P × t

Problem 57

During a workout, football players ran up the stadium stairs in 75 s. The distance along the stairs is 83 m and they are inclined at a 33° angle. If a player has a mass of 82 kg, estimate his average power output on the way up. Ignore friction and air resistance.