# Chapter 4:

## Dynamics: Newton's Laws of Motion

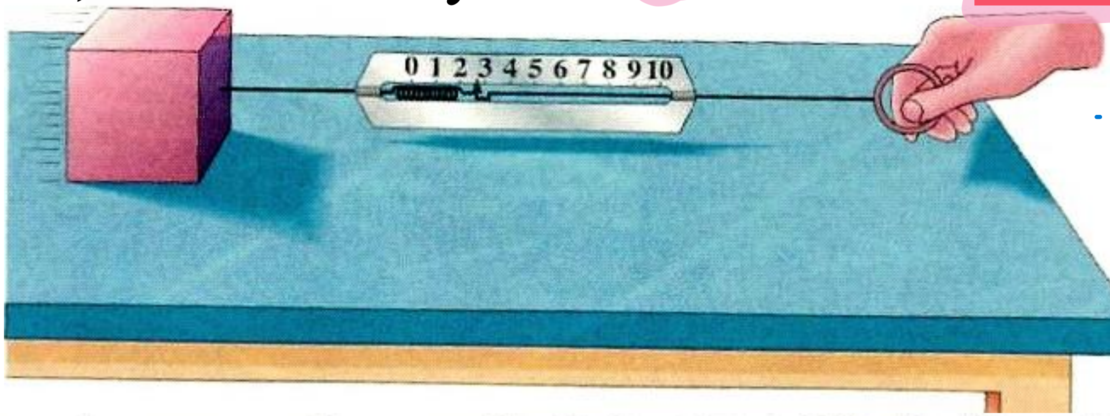
Dynamics

why

- *Department of Physics*
- *The University of Jordan*

# Force

A ***Force*** is "A push or a pull" on an object. Usually, for a force, we use the symbol **F.** **F is a *VECTOR*!**




One way to measure the magnitude (or strength) of a force is to make use of a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the body (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

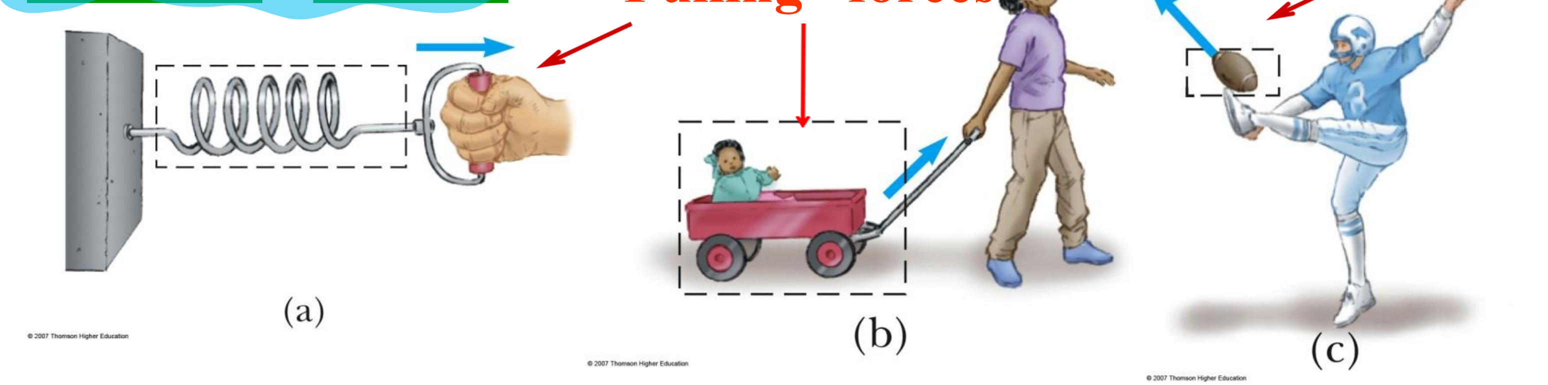
***Obviously, vector addition is needed to add forces!***

# Classes of Forces

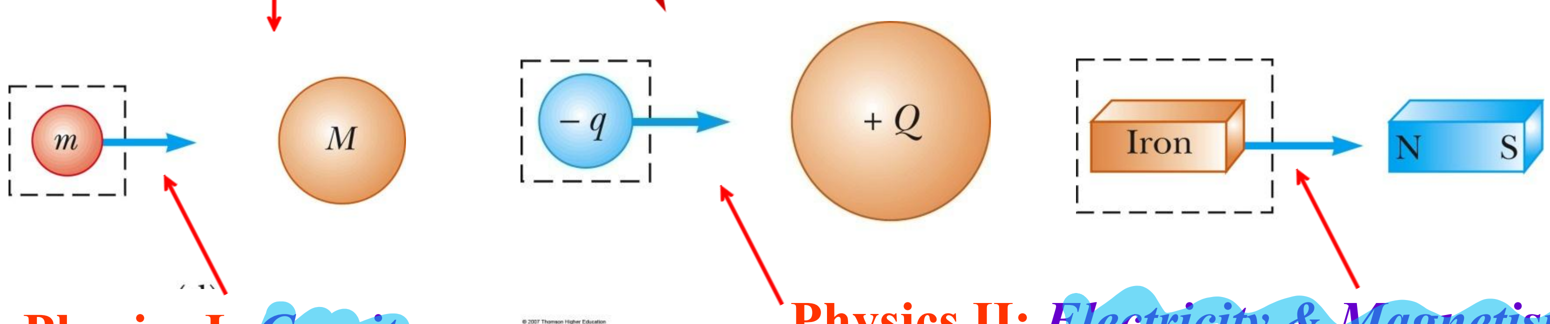
## "Contact" Forces:

"Pulling" forces

"Pushing" forces




## "Field" Forces:




Physics I: *Gravity*

Physics II: *Electricity & Magnetism*

# Newton's First Law

Newton was born the same year Galileo died!

- ***Newton's First Law*** (The "Law of Inertia"):

**"Every object continues in a state of rest or uniform motion (constant velocity) in a straight line unless acted on by a net force."**

# Newton's First Law of Motion

## Inertial Reference Frames

### Newton's 1st Law:

- Doesn't hold in every reference frame. In particular, it doesn't work in ***such a reference frame that is accelerating or rotating.***

***An Inertial Reference frame is one in which Newton's first law is valid.***

- This excludes rotating & accelerating frames.
- How can we tell if we are in an inertial reference frame?

***By checking to see if Newton's First Law holds!***

# Conceptual Example 4-1:

## **Newton's First Law**.

A school bus comes to a sudden stop, and all of the backpacks on the floor start to slide forward.

***What force causes them to do this?***

# Inertia & Mass

- ***Inertia*** **≡** The tendency of a body to maintain its state of rest or motion.
- ***MASS*** **≡** A measure of the inertia of a body.
  - The quantity of matter in a body.
  - The SI System quantifies mass by having a standard mass **= Standard Kilogram (kg)**

    (Similar to the standards for length & time).
  - The SI Unit of Mass = **The Kilogram (kg)**
    - The cgs unit of mass = the gram (g) = $10^{-3}$ kg
- **Weight is *NOT*** the same as mass!
  - Weight is the force of gravity on an object.
    - Discussed later in the chapter.

# Newton's Second Law (Lab)

- ***Newton's 1st Law*:** If no net force acts, an object remains at rest or in uniform motion in straight line.
- What if a net force acts? That question is answered by doing

***Experiments*.**

- It is found that, if **the net force** $\sum \mathbf{F} \neq 0 \Rightarrow$

  The velocity **v** changes (in magnitude, in direction or both).
- A change in the velocity **v** ($\Delta$**v**).

  $\Rightarrow$ There is an acceleration $\mathbf{a} = (\Delta \mathbf{v}/\Delta t)$ ***OR***

***A net force acting on a body produces an acceleration!***

$$\sum \mathbf{F} \Rightarrow \mathbf{a}$$

# Newton's 2nd Law

## Experiments Show That:

The net force $\sum F$ on a body & the acceleration **a** of that body are related.

- How are they related? Answer this by doing more

***EXPERIMENTS!***

  - **Thousands of experiments** over **hundreds of years** find (for an object of mass **m**): $a \propto \sum F/m$ (proportionality)

- The SI system chooses the units of force so that this is not just a proportionality but an

***Equation*:** $a \equiv \sum(F/m)$ ***OR*** **(total force!)** $\rightarrow$

$$F_{net} \equiv \sum F = ma$$

# ***Newton's 2nd Law***: $F_{net} = ma$

$F_{net}$ = the net (***TOTAL!***) force acting on mass **m**

**m** = mass (inertia) of the object. **a** = acceleration of the object.

***OR***, **a** = a description of the ***effect of F***.

***OR***, ***F is the cause of a***.

- To emphasize that **F** in ***Newton's 2nd Law*** is the ***TOTAL*** **(net) force** on the mass **m**, your text writes:

$$\sum F = ma$$

≡ ***The Vector Sum*** **of *all Forces* on mass m!**

$\sum$ = a math symbol meaning **sum** (capital sigma)

- ***Newton's 2nd Law*:**

**Based on experiment!**

***Not derivable mathematically!!***

$$\sum F = ma$$

***A VECTOR*** **Equation!!**

It holds component by component.

$$\sum F_x = ma_x, \sum F_y = ma_y, \sum F_z = ma_z$$

**THIS IS ONE OF THE**

***MOST FUNDAMENTAL & IMPORTANT***

***LAWS OF CLASSICAL PHYSICS!!!***

# Summary

$$\sum \vec{F} = m\vec{a}$$

- **Newton's 2nd Law** is the relation between acceleration & force.
- Acceleration is proportional to force and inversely proportional to mass.
- ***It takes a force to change either the direction of motion or the speed of an object.***
- More force means more acceleration; the same force exerted on a more massive object will yield less acceleration.

Now, ***a more precise definition of Force*:**

**Force ≡ *An action capable of accelerating an object*.**

Force is a vector & $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ is true along each coordinate axis.

**TABLE 4–1**
**Units for Mass and Force**

| System | Mass | Force |
|---|---|---|
| SI | kilogram (kg) | newton (N) $(= \text{kg} \cdot \text{m/s}^2)$ |
| cgs | gram (g) | dyne $(= \text{g} \cdot \text{cm/s}^2)$ |
| British | slug | pound (lb) |

Conversion factors: 1 dyne $= 10^{-5}$ N; 1 lb $\approx$ 4.45 N.

The **SI unit of force** is

**The Newton (N)**

$\sum F = ma$, unit = kg m/s$^2$

$\Rightarrow$ **1N = 1 kg m/s$^2$**

**Note**

The pound is a unit of force, not of mass, & can therefore be equated to Newtons but not to kilograms.

# Examples

## Example 4-2:

Estimate the net force needed to accelerate

**(a)** a **1000-kg** car at **a = (½)*g***

**(b)** a **200-g apple** at the same rate.

**SOLUTION** (*a*) The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\,\mathrm{m/s^2}) \approx 5\,\mathrm{m/s^2}$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma \approx (1000\,\mathrm{kg})(5\,\mathrm{m/s^2}) = 5000\,\mathrm{N}.$$

(*b*) For the apple, $m = 200\,\mathrm{g} = 0.2\,\mathrm{kg}$, so

$$\Sigma F = ma \approx (0.2\,\mathrm{kg})(5\,\mathrm{m/s^2}) = 1\,\mathrm{N}.$$

# Examples

## Example 4-3:

Force to stop a car.

What average net force is required to bring a **1500-kg** car to rest from a speed of **100 km/h (27.8 m/s)** within **3.97 s** ?


**Solution:**

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 - 27.8}{3.97} = -7\ m/s^2$$

$$\sum F = ma = (1500)(-7) = -10500\ N = 1.1x10^4 N$$

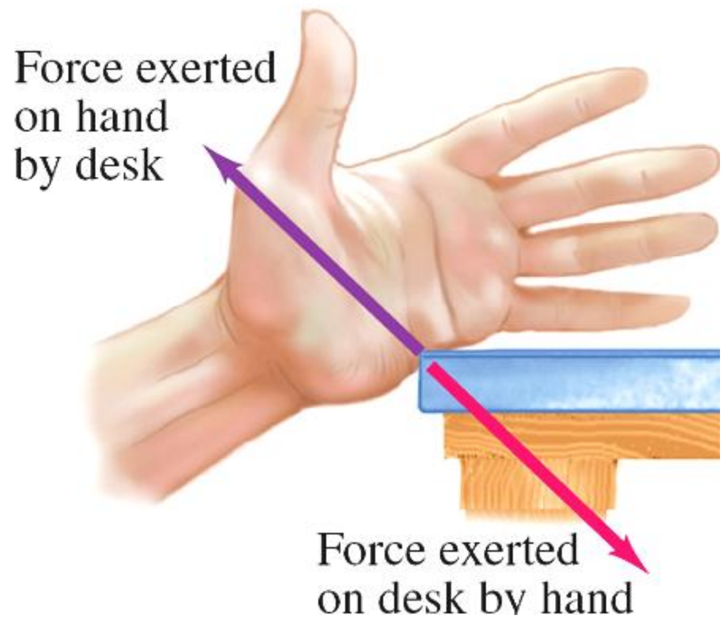
average net force 10500 to the left

# Newton's 3rd Law

"Whenever one object exerts a force **F** on a second object, the second object exerts an ***equal and opposite*** force **-F** on the first object."

– **Law of Action-Reaction:** "Every action has an equal & opposite reaction". (Note that action-reaction forces act on ***DIFFERENT*** objects!)

If your hand pushes against the edge of a desk (**red force vector**), the desk pushes back against your hand (**purple force vector**; the two colors tell us that this force acts on a ***DIFFERENT*** object).

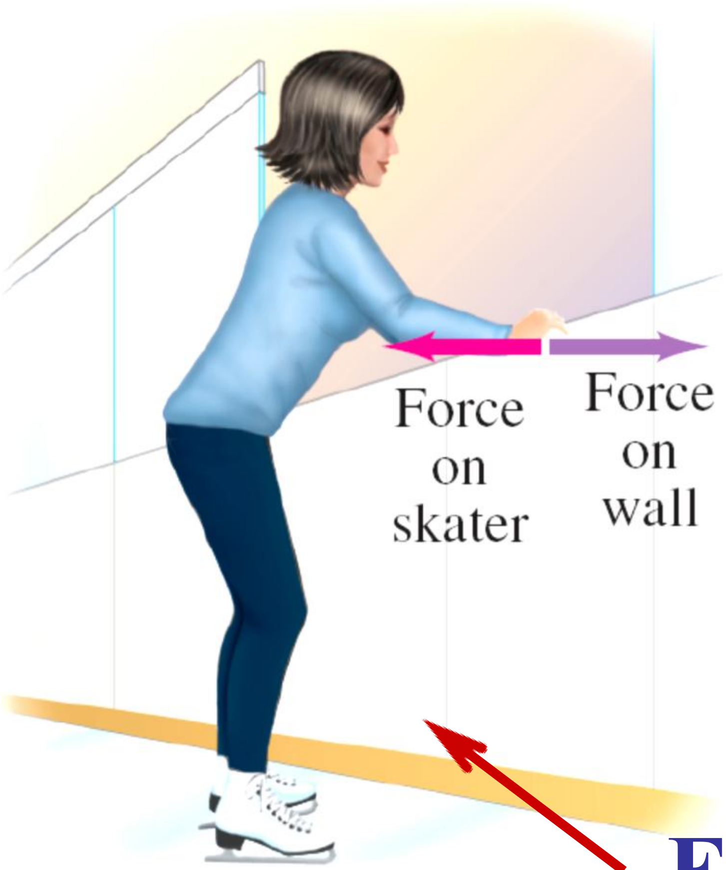


# Newton's 3rd Law

## Alternative Statements

1. Forces ***ALWAYS*** occur in pairs
2. A single isolated force ***CANNOT*** exist
3. The "action force" is equal in magnitude to the "reaction force" & opposite in direction.
   a. One of the forces is the "action force", the other is the "reaction force"
   b. It doesn't matter which is considered the "action" & which the "reaction"
   c. The action & reaction forces ***MUST ACT ON DIFFERENT OBJECTS***.

# Action-Reaction Pairs: Act on *Different* Objects




The key to correct application of

**Newton's 3rd Law** is:

***THE FORCES ARE EXERTED ON DIFFERENT OBJECTS.***

Make sure you don't use them as if they were acting on the ***same*** object.

## Example

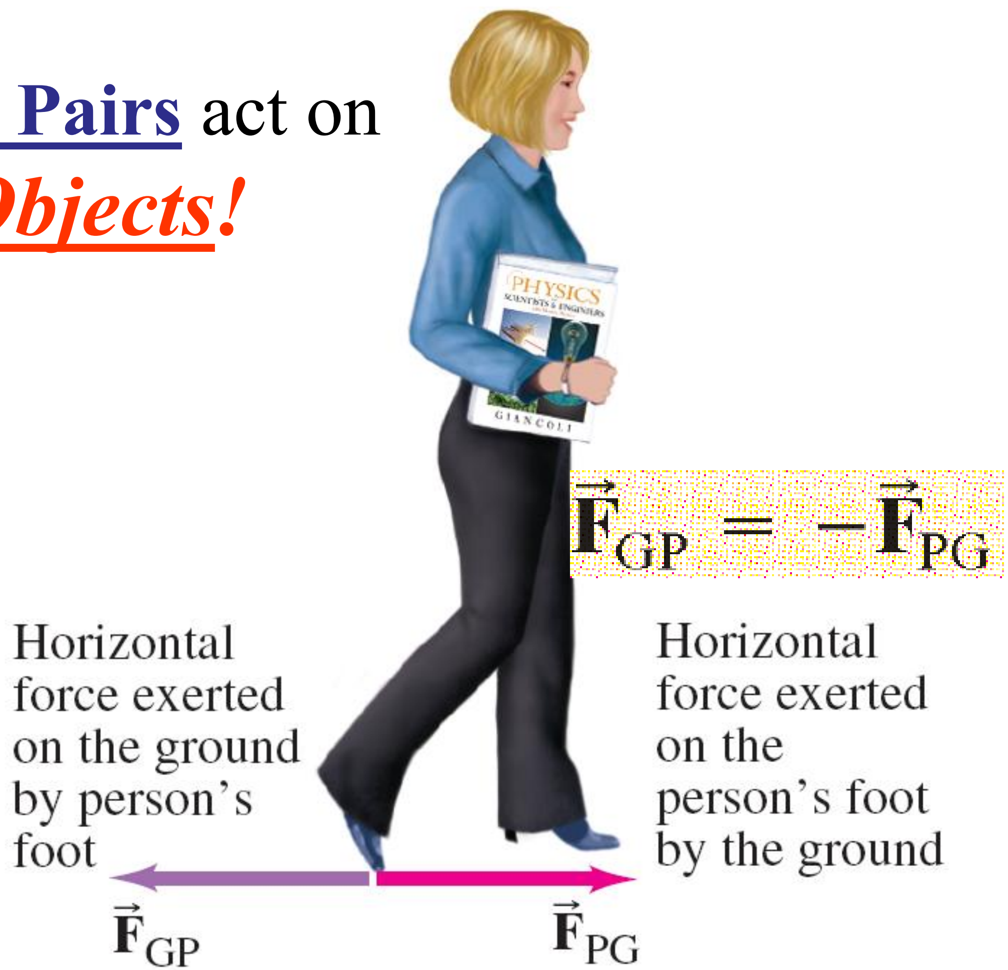
An ice skater pushes against a railing. The railing pushes back & this force causes her to move away.

# Helpful Notation

On forces, the 1st subscript is the object that the force is being exerted on; the 2nd is the source.

**Action-Reaction Pairs** act on

***Different Objects!***




$$\vec{F}_{GP} = -\vec{F}_{PG}$$

# Sect. 4-6: Weight & Normal Force

***Weight*** ≡ **The force of gravity on an object.**

- Write as $\mathbf{F_G} \equiv \mathbf{W}$.
- Consider an object in free fall.

**Newton's 2nd Law** is:

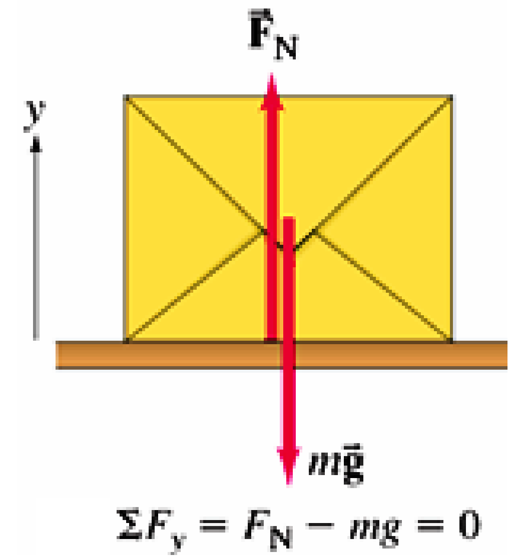
$$\sum \mathbf{F} = \mathbf{ma}$$

- If no other forces are acting, only $\mathbf{F_G}$ ($\equiv$ **W**) acts (in the vertical direction). $\sum \mathbf{F_y} = \mathbf{ma_y}$

  Or: $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, (down, of course)
- **SI Units**: **Newtons** (just like any force!).

$$\mathbf{g = 9.8\ m/s^2} \Rightarrow \text{If } \mathbf{m = 1\ kg,\ W = 9.8\ N}$$

# "Normal" Force




- Suppose an object is at rest on a table. No motion, but does the force of gravity stop? ***OF COURSE NOT!***
- But, the object does not move:

  $2^{nd}$ Law $\Rightarrow$ $\sum$**F = ma = 0**

  $\Rightarrow$ There must be ***some other force*** acting besides gravity (weight) to have $\sum$**F = 0.**
- That force $\equiv$ ***The Normal Force*** $\mathbf{F_N}$ **(= N)**

  "Normal" is a math term for perpendicular ($\perp$)

  $\mathbf{F_N}$ is $\perp$ to the surface & opposite to the weight

  (**in this simple case *only***!) ***Caution!!!***

  $\mathbf{F_N}$ ***isn't always = & opposite to the weight,*** as we'll see!

# Normal Force

- **Where does the normal force come from?**

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- **From the *<u>other object</u>*!!!**
- **Is the normal force *ALWAYS* equal & opposite to the weight?**

***NO!!!***

**Example:** Find the acceleration (**a**) and the Normal Force $\mathbf{F_N}$

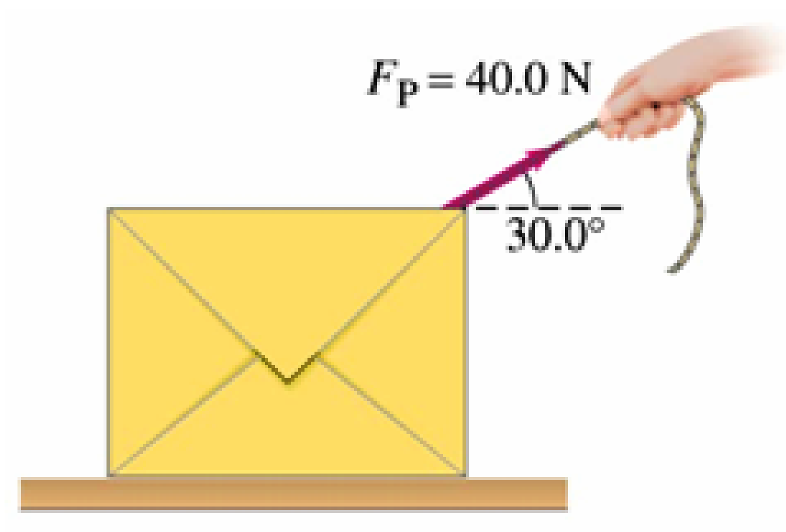
**The normal force is *NOT* always equal & opposite to the weight!!**

# Example 4-11

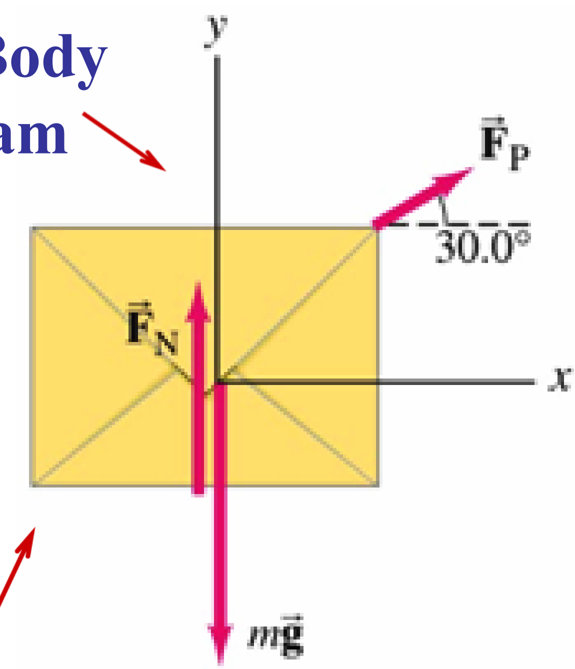
**A box of mass m = 10 kg is pulled by an attached cord along a horizontal smooth (frictionless!) surface of a table. The force exerted is $F_P = 40.0$ N at a 30.0° angle as shown. Calculate:**

**a. The acceleration of the box.**

**b. The magnitude of the upward normal force $F_N$ exerted by the table on the box.**



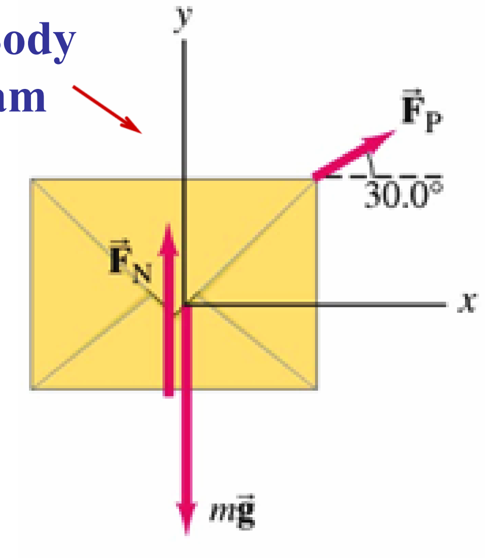

**Free Body Diagram**




**The normal force, $F_N$ is *NOT* always equal & opposite to the weight!!**

# Example 4-11

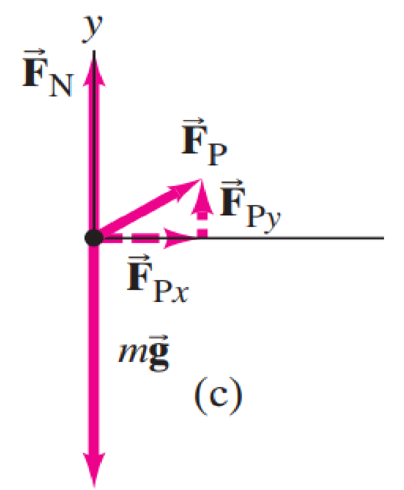
Free Body Diagram




$$F_{Px} = (40.0\,\text{N})(\cos 30.0°) = (40.0\,\text{N})(0.866) = 34.6\,\text{N},$$
$$F_{Py} = (40.0\,\text{N})(\sin 30.0°) = (40.0\,\text{N})(0.500) = 20.0\,\text{N}.$$

$$F_{Px} = ma_x.$$

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\text{N})}{(10.0\,\text{kg})} = 3.46\,\text{m/s}^2.$$




$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

$$F_N = 78.0\,\text{N}.$$

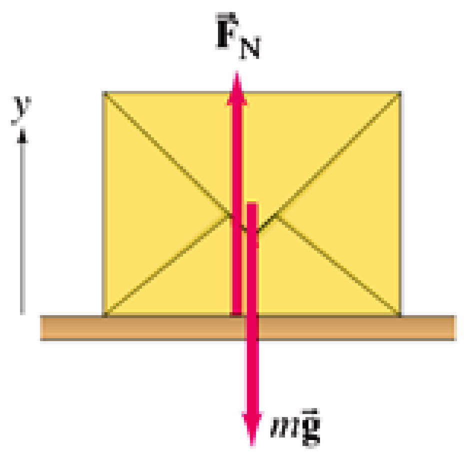
# Example 4-6

**EXAMPLE 4–6** **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–15a). (*a*) Determine the weight of the box and the normal force exerted on it by the table. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–15b. Again determine the normal force exerted on the box by the table. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–15c), what now is the normal force exerted on the box by the table?

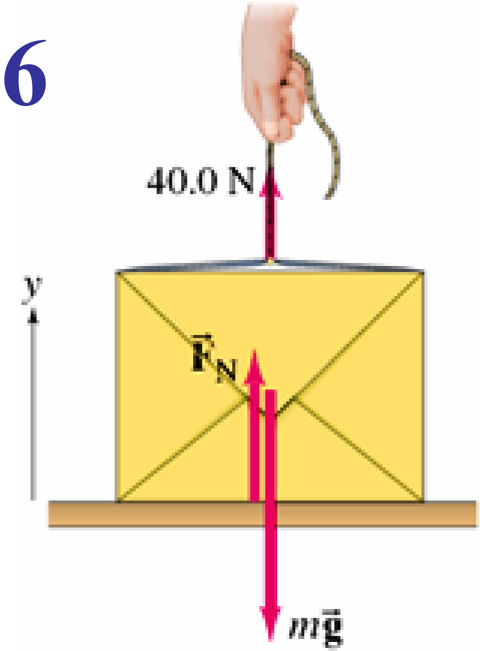
**m = 10 kg**

**The normal force is *NOT* always equal & opposite to the weight!!**

# Example 4-6

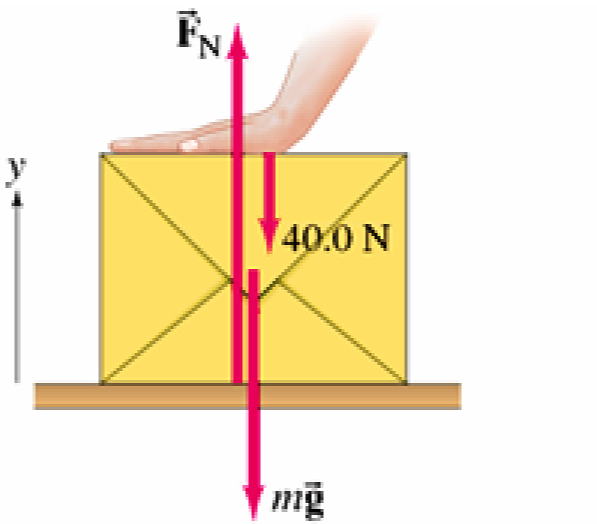



(a) $\Sigma F_y = F_N - mg = 0$




(c) $\Sigma F_y = F_N - mg + 40.0\text{ N} = 0$

$F_N = mg$ **= 98.0 N upward**




(b) $\Sigma F_y = F_N - mg - 40.0\text{ N} = 0$

$$F_N = mg - 40.0\text{ N} = 98.0\text{ N} - 40.0\text{ N} = 58.0\text{ N}.$$

The table does not push against the full weight of the box because of the upward force exerted by your friend.

$$F_N = mg + 40.0\text{ N} = 98.0\text{ N} + 40.0\text{ N} = 138.0\text{ N}$$

# Example 4-7

What happens when a person pulls upward on the box in the previous example with a force greater than the box's weight, say **100.0 N**?

***The box will accelerate upward because***

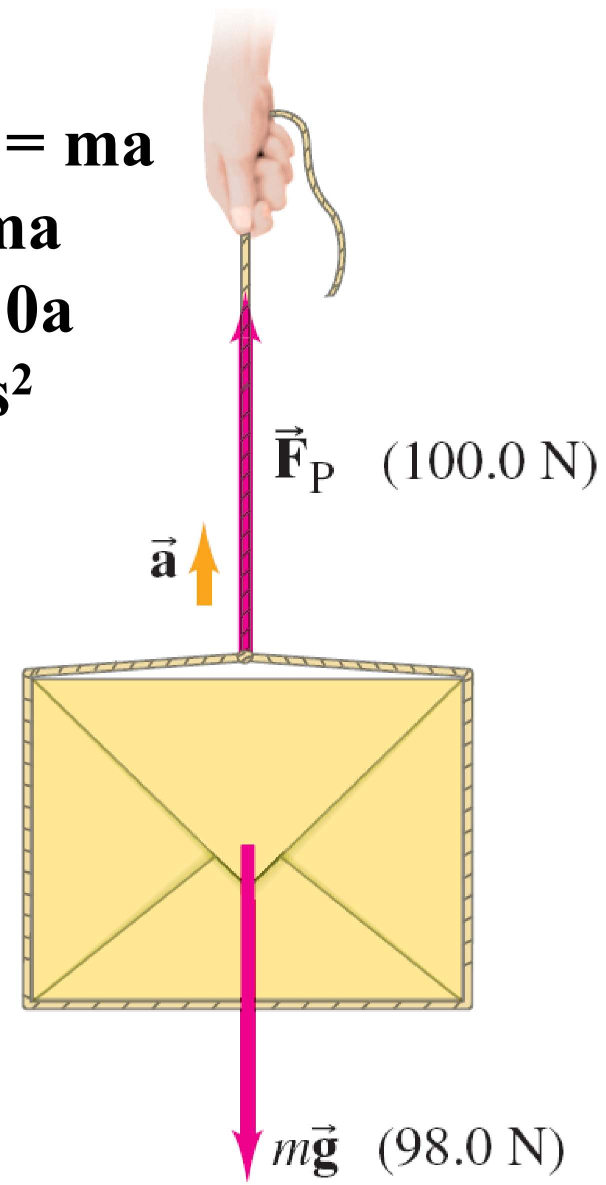
$F_P > mg$!!

**m = 10 kg,** $\sum F = ma$

$$F_P - mg = ma$$

$$100 - 98 = 10a$$

$$a = 0.2\ \text{m/s}^2$$



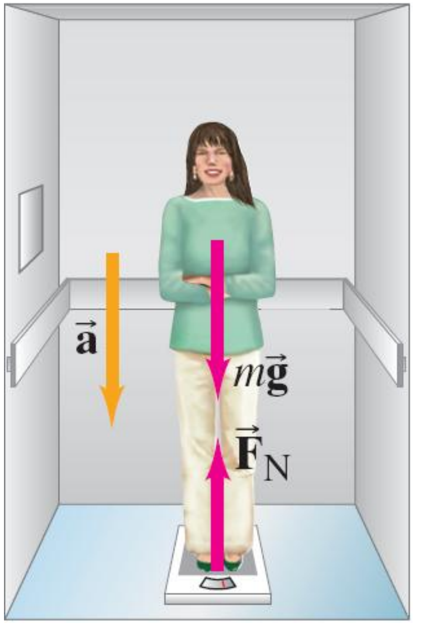

**Note:** ***The normal force is zero here*** because the mass isn't in contact with a surface!

# Example 4-8: Apparent "weight loss"

A **65-kg** woman descends in an elevator that accelerates at **0.20g** downward. She stands on a scale that reads in **kg**.

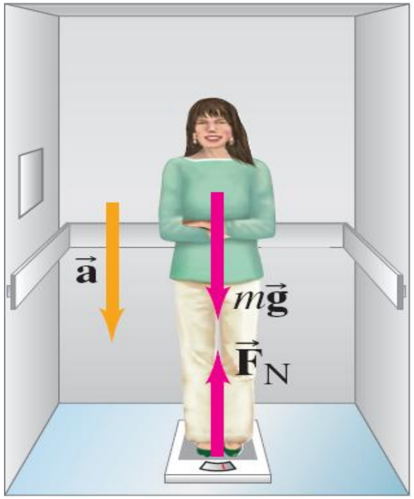
**(a)** During this acceleration, what is her weight & what does the scale read?

**(b)** What does the scale read when the elevator descends at a constant speed of **2.0 m/s**?



**SOLUTION** (*a*) From Newton's second law,

$$\begin{aligned} \Sigma F &= ma \\ mg - F_N &= m(0.20g). \end{aligned}$$




We solve for $F_N$:

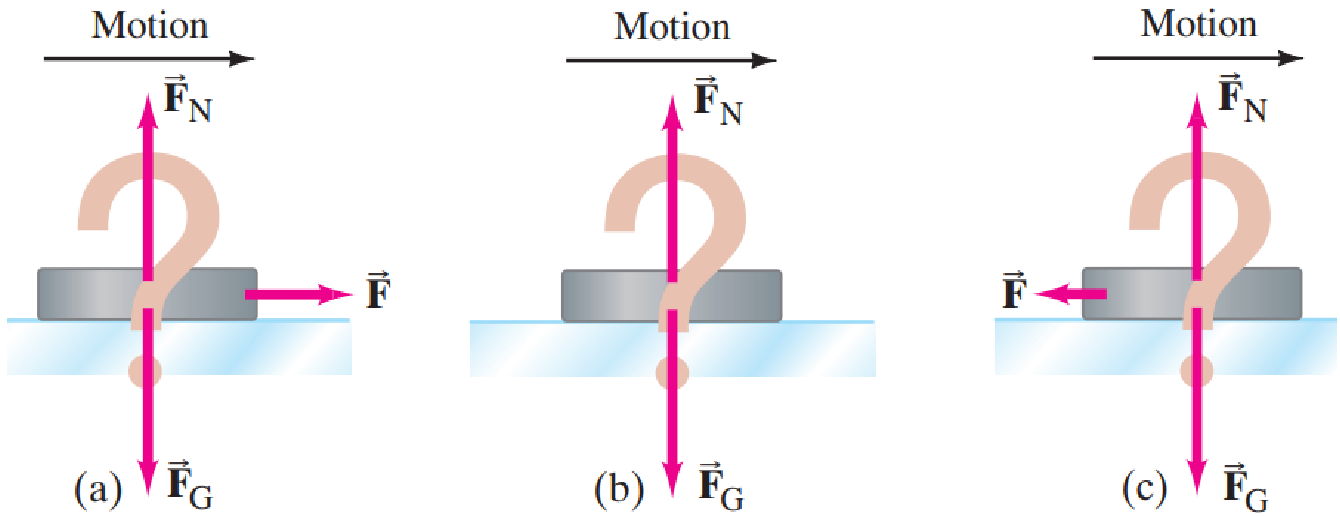
$$\begin{aligned} F_N &= mg - 0.20mg \\ &= 0.80mg, \end{aligned}$$

and it acts upward. The normal force $\vec{F}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\,\text{kg})(9.8\,\text{m/s}^2) = 640\,\text{N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\,\text{kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

# Conceptual Example 4-10

**Moving at constant v, with *NO friction*, which free body diagram is correct?**

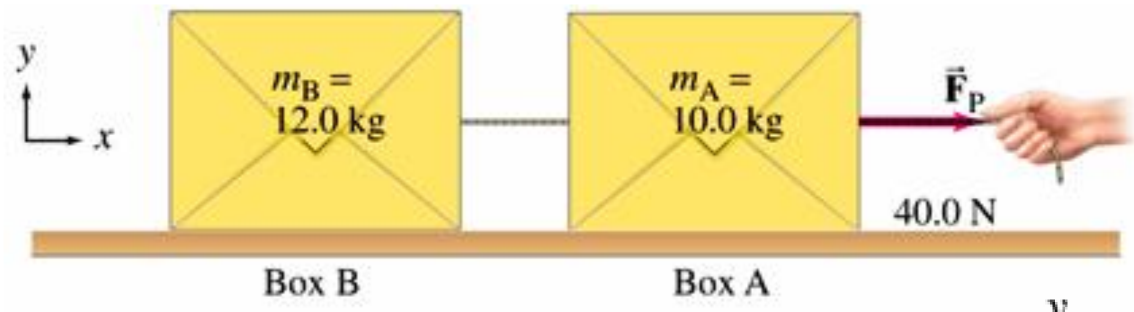


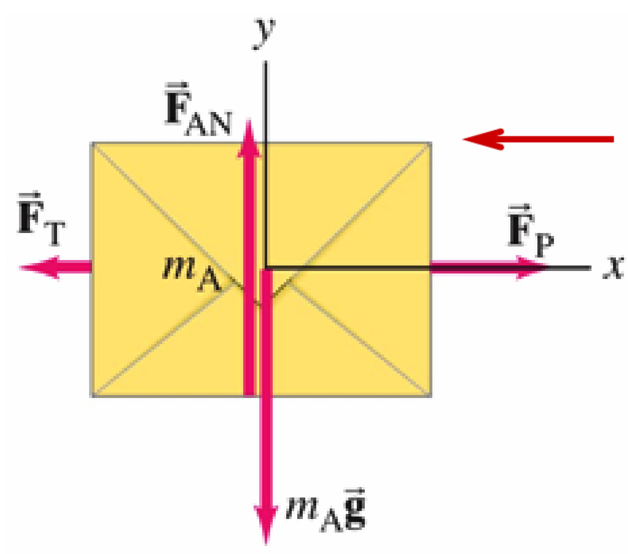

**It is (b) that is correct**

# Example 4-12

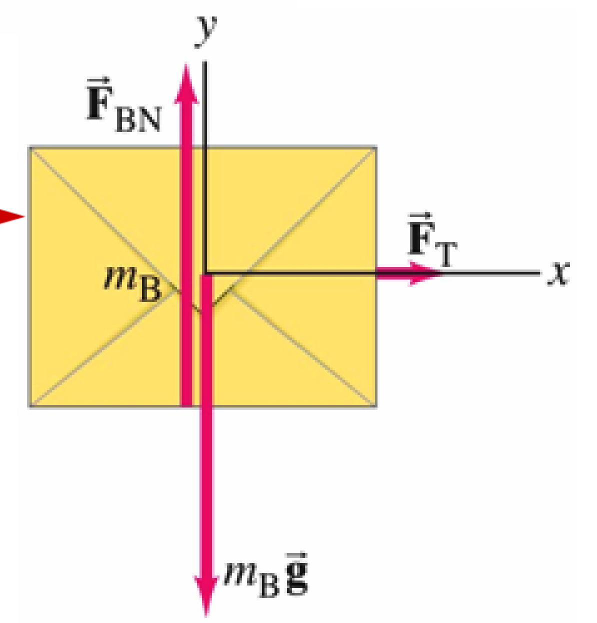
**Two boxes are connected by a lightweight (massless!) cord & are resting on a smooth (frictionless!) table. The masses are $m_A$ = 10 kg & $m_B$ = 12 kg. A horizontal force $F_P$ = 40 N is applied to $m_A$. Calculate:**

**a. Acceleration of the boxes. b. Tension in the cord connecting the boxes.**

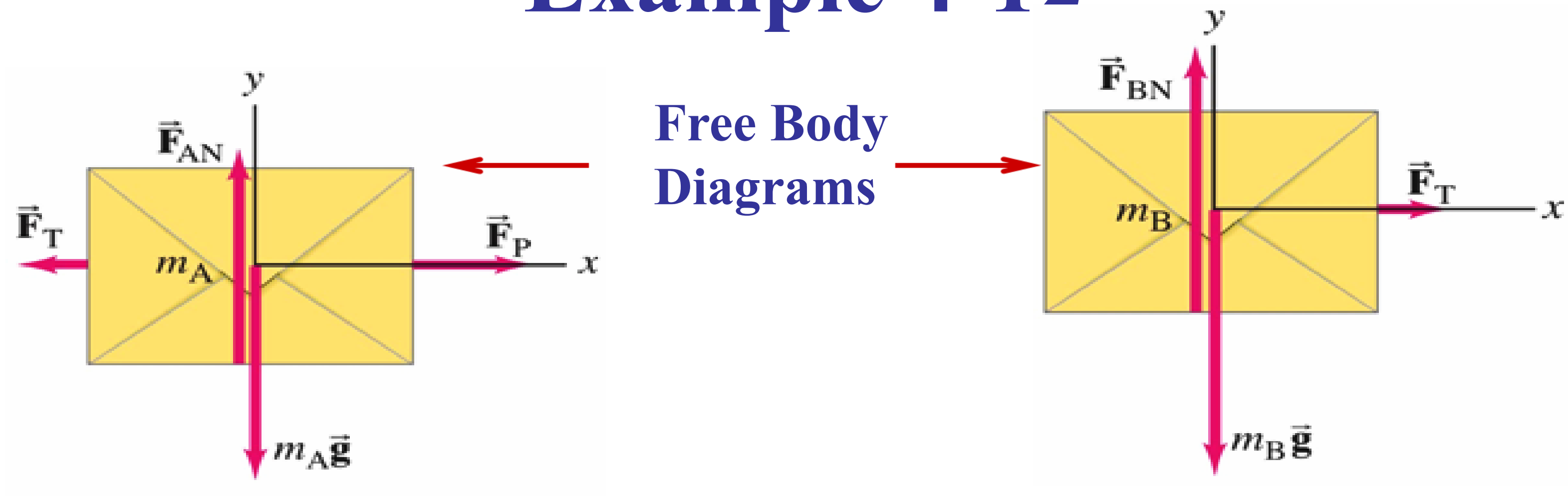



**Free Body Diagrams**



# Example 4-12




**SOLUTION** (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is $F_T$, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

$$a_A = a_B = a.$$

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

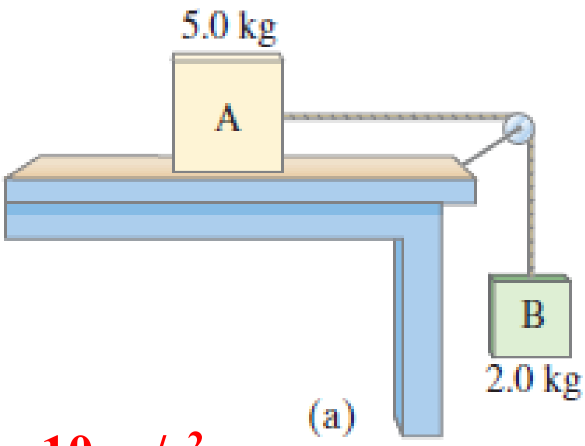
$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\,\text{N}}{22.0\,\text{kg}} = 1.82\,\text{m/s}^2.$$

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\,\text{kg})(1.82\,\text{m/s}^2) = 21.8\,\text{N}.$$

Thus, $F_T < F_P$ $(= 40.0\,\text{N})$, as we expect, since $F_T$ acts to accelerate only $m_B$.

# Example: Two boxes and a pulley.




Find 1. The acceleration of Box B and Box A

2. The Tension in the cord

3. The normal force on A and B ? Use $g = 10\ m/s^2$

# Conceptual Example 4-14

**Advantage of a Pulley**

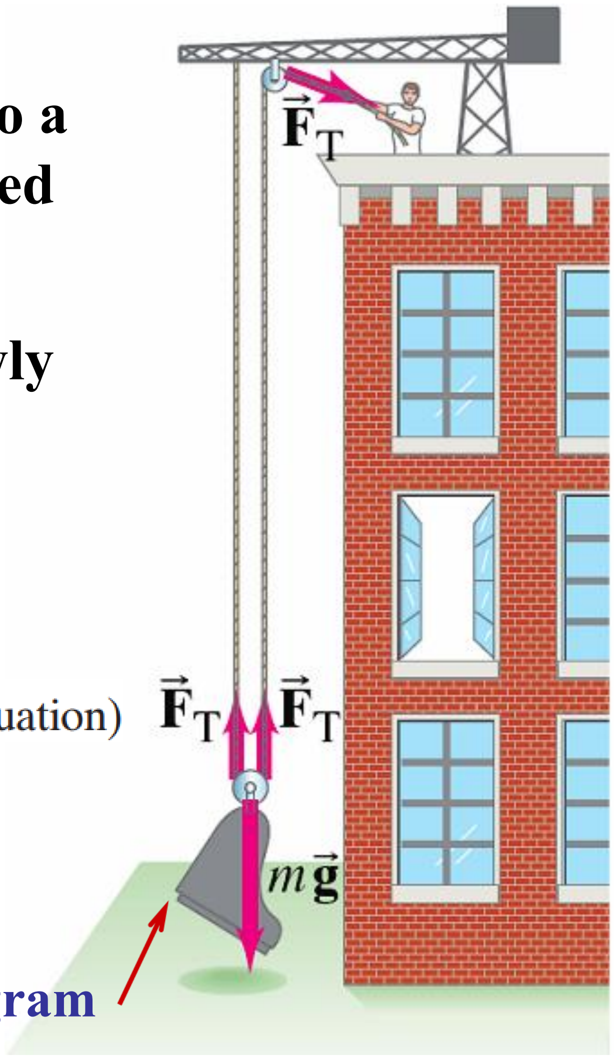
**A mover is trying to lift a piano (slowly) up to a second-story apartment. He uses a rope looped over 2 pulleys.**

**What force must he exert on the rope to slowly lift the piano's mg = 2000-N weight?**

$$2F_T - mg = ma.$$

To move the piano with constant speed (set $a = 0$ in this equation)

$$F_T = mg/2.$$




Free Body Diagram

# General Approach to Problem Solving

1. **Read** the problem carefully; then read it again.
2. **Draw** a sketch, then a free-body diagram.
3. **Choose** a convenient coordinate system.
4. **List** the known & unknown quantities; **find** relationships between the knowns & the unknowns.
5. **Estimate** the answer.
6. **Solve** the problem without putting in any numbers (algebraically); once you are satisfied, put the numbers in.
7. **Keep track of dimensions.**
8. ***Make sure your answer is REASONABLE!***

# Problem 25

**25.** (II) One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Fig. 4–49. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of 1.25 $m/s^2$ by the upper cord, calculate the tension in each cord.

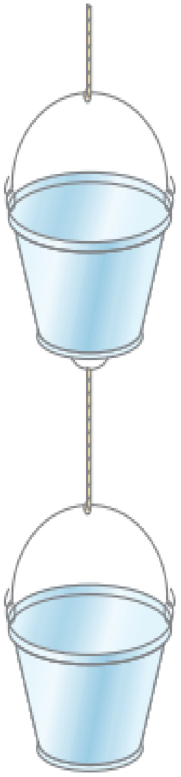


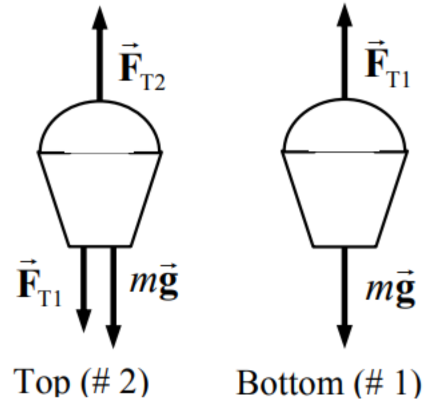
**FIGURE 4–49**
Problem 25.

# Problem 25

**$\sum F = ma$ (y direction)**

**for *EACH* bucket separately!!!**

**Take up as positive.**




(*a*) Since the buckets are at rest, their acceleration is 0. Write Newton's second law for each bucket, calling UP the positive direction.

$$\sum F_1 = F_{T1} - mg = 0 \quad \rightarrow$$

$$F_{T1} = mg = (3.2\text{ kg})(9.80\text{ m/s}^2) = \boxed{31\text{ N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = 0 \quad \rightarrow$$

$$F_{T2} = F_{T1} + mg = 2\,mg = 2(3.2\text{ kg})(9.80\text{ m/s}^2) = \boxed{63\text{ N}}$$

(*b*) Now repeat the analysis, but with a nonzero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{T1} - mg = ma \quad \rightarrow$$

$$F_{T1} = mg + ma = (3.2\text{ kg})(9.80\text{ m/s}^2 + 1.25\text{ m/s}^2) = 35.36\text{ N} \approx \boxed{35\text{ N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = ma \quad \rightarrow \quad F_{T2} = F_{T1} + mg + ma = 2F_{T1} = \boxed{71\text{ N}}$$

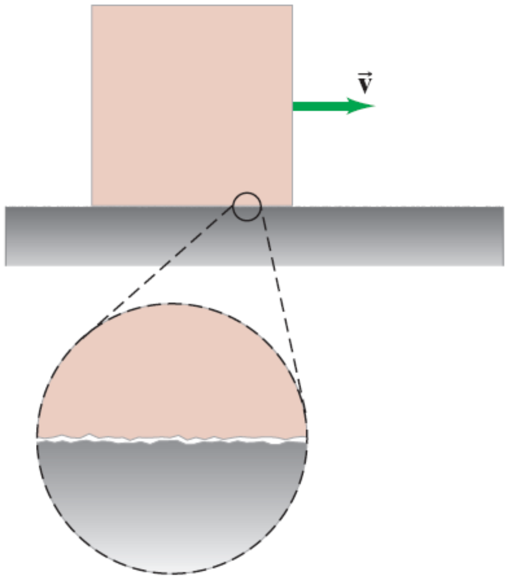
# Friction

- ***Friction*** ***is always*** **present when 2 solid surfaces slide along each other.**
  - We must account for it to be realistic!
  - Exists between any 2 sliding surfaces.
  - There are two types of friction:

    **Static** (no motion) friction

    **Kinetic** (motion) friction

  - The size of the friction force depends on the microscopic details of 2 sliding surfaces. These details aren't fully understood & depend on
    - The materials they are made of
    - Are the surfaces smooth or rough?
    - Are they wet or dry?
    - Etc., etc., etc.

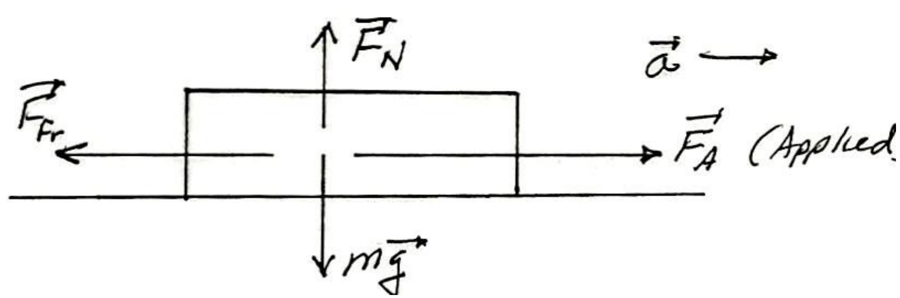


- **Kinetic Friction** is the same as ***sliding friction***. Experiments determine the relation used to calculate friction force.
- The **kinetic friction** force $\mathbf{F_{fr}}$ is proportional to the ***magnitude*** of the normal force $\mathbf{F_N}$ between 2 sliding surfaces.

  The **DIRECTIONS** of $\mathbf{F_{fr}}$ & $\mathbf{F_N}$ are ⊥ each other!! $\mathbf{F_{fr}} \perp \mathbf{F_N}$
- We write their relation as $\mathbf{F_{fr}} \equiv \mu_k \mathbf{F_N}$ (**magnitudes**)

**$\mu_k \equiv$ Coefficient of Kinetic Friction**

$\mu_k$ depends on the surfaces & their conditions, is dimensionless & < 1 It is different for each pair of sliding surfaces.

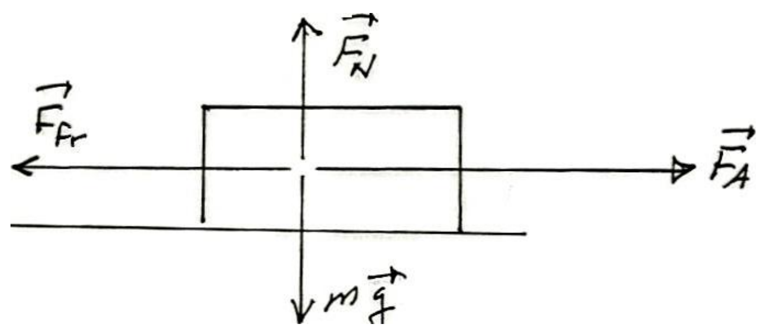


- **Static Friction:** A friction force $\mathbf{F_{fr}}$ exists || 2 surfaces, even if there is no motion. Applies when 2 surfaces are at rest with respect to each other. The static friction force is as big as it needs to be to prevent slipping, up to a maximum value. Usually it is easier to keep an object sliding than it is to get it started. Consider an applied force $\mathbf{F_A}$:

$$\sum \mathbf{F} = m\mathbf{a} = 0 \ \& \text{ also } \mathbf{v} = 0$$

$\Rightarrow$ ***There must be a friction force*** $\mathbf{F_{fr}}$ to oppose $\mathbf{F_A}$

$$\Rightarrow \mathbf{F_A} - \mathbf{F_{fr}} = 0 \text{ or } \mathbf{F_{fr}} = \mathbf{F_A}$$



- Experiments find that ***the maximum static friction force*** $\mathbf{F_{fr}}$ **(max)** is proportional to the **magnitude** (size) of the normal force $\mathbf{F_N}$ **between** the 2 surfaces. The ***DIRECTIONS*** of $\mathbf{F_{fr}}$ & $\mathbf{F_N}$ are ⊥ each other!! $\mathbf{F_{fr} \perp F_N}$
- Write the relation as $\mathbf{F_{fr}}$ **(max) =** $\mathbf{\mu_s F_N}$ (magnitudes)

$$\mu_s \equiv \textbf{Coefficient of static friction}$$

  - Depends on the surfaces & their conditions
  - Dimensionless & < 1
  - ***Always*** find $\mu_s > \mu_k$

$\Rightarrow$ Static friction force: $\mathbf{F_{fr} \leq \mu_s F_N}$

# Coefficients of Friction

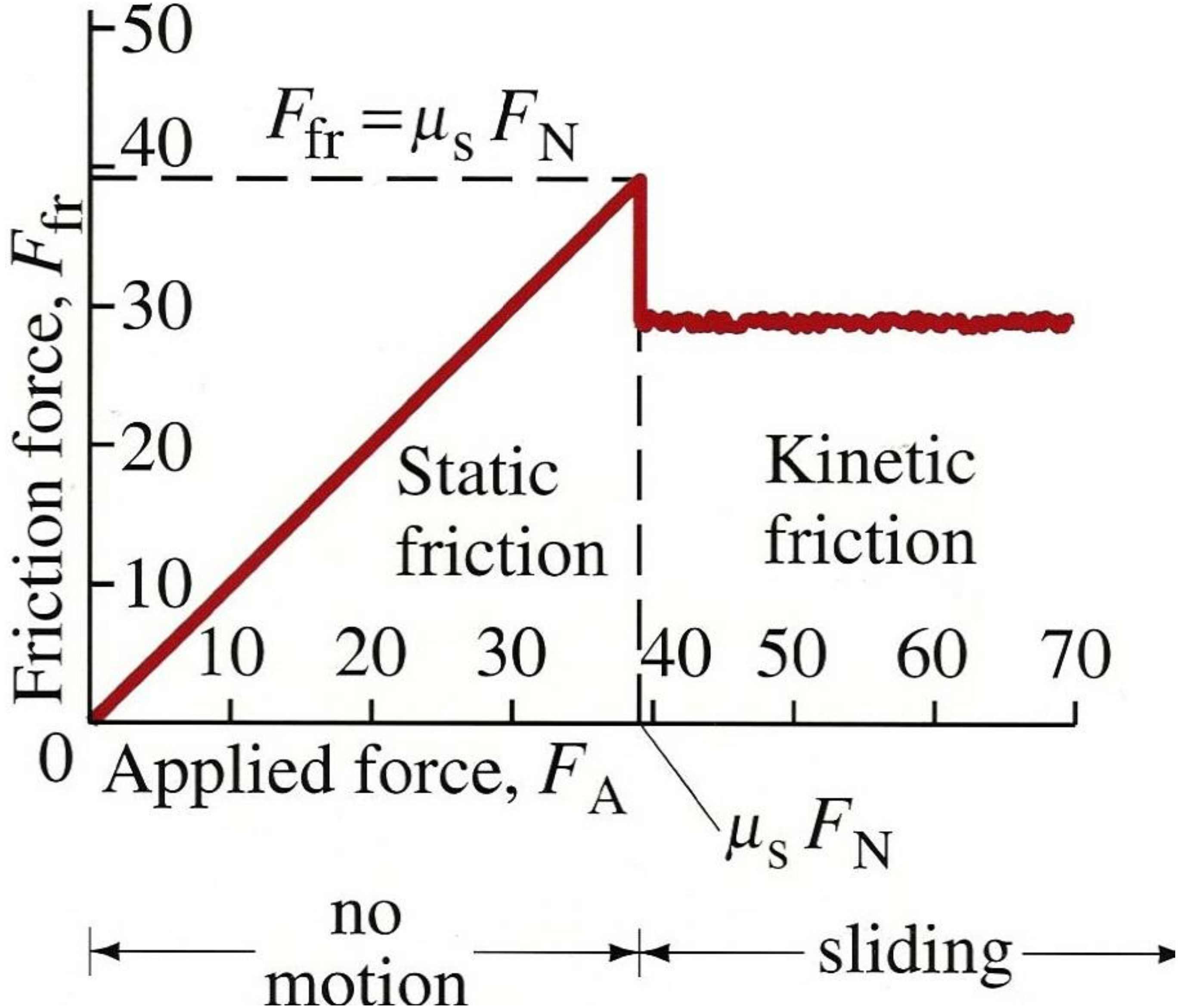
**TABLE 4–2 Coefficients of Friction**[†]

| Surfaces | Coefficient of Static Friction, $\mu_s$ | Coefficient of Kinetic Friction, $\mu_k$ |
|---|---|---|
| Wood on wood | 0.4 | 0.2 |
| Ice on ice | 0.1 | 0.03 |
| Metal on metal (lubricated) | 0.15 | 0.07 |
| Steel on steel (unlubricated) | 0.7 | 0.6 |
| Rubber on dry concrete | 1.0 | 0.8 |
| Rubber on wet concrete | 0.7 | 0.5 |
| Rubber on other solid surfaces | 1–4 | 1 |
| Teflon® on Teflon in air | 0.04 | 0.04 |
| Teflon on steel in air | 0.04 | 0.04 |
| Lubricated ball bearings | $<0.01$ | $<0.01$ |
| Synovial joints (in human limbs) | 0.01 | 0.01 |

[†] Values are approximate and intended only as a guide.

$\mu_s > \mu_k \Rightarrow F_{fr}$ (max, static) $> F_{fr}$ (kinetic)

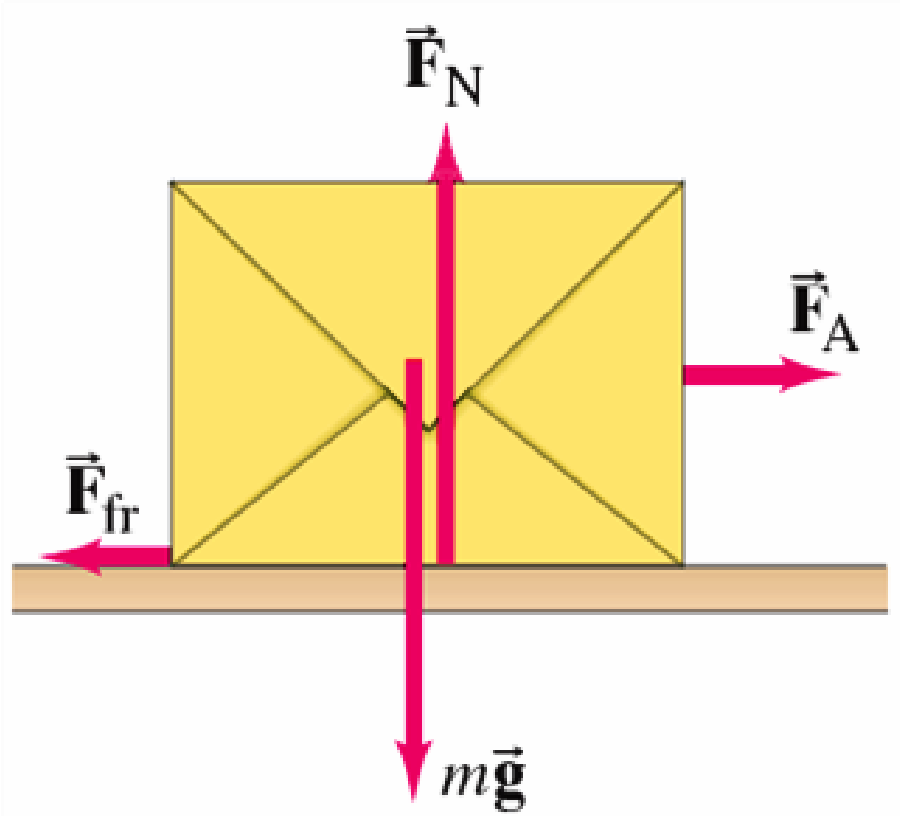
# Static & Kinetic Friction



# Example 4-16: Friction; Static & Kinetic

**A box, mass m =10.0-kg rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.4$; the coefficient of kinetic friction is $\mu_k = 0.3$. Calculate the friction force on the box for a horizontal external applied force of magnitude:**

**(a) 0, (b) 10 N, (c) 20 N, (d) 38 N, (e) 40 N.**



# Example 4-16: Friction; Static & Kinetic

$\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0.$

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.

**(b), (c), and (d) the applied force is not sufficient to move the box**

$$\Sigma F_x = F_A - F_{fr} = 0$$

$\boldsymbol{F_{fr}(b) = 10\ \text{N}}$ $\boldsymbol{F_{fr}(c) = 20\ \text{N}}$ $\boldsymbol{F_{fr}(d) = 38\ \text{N}}$

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

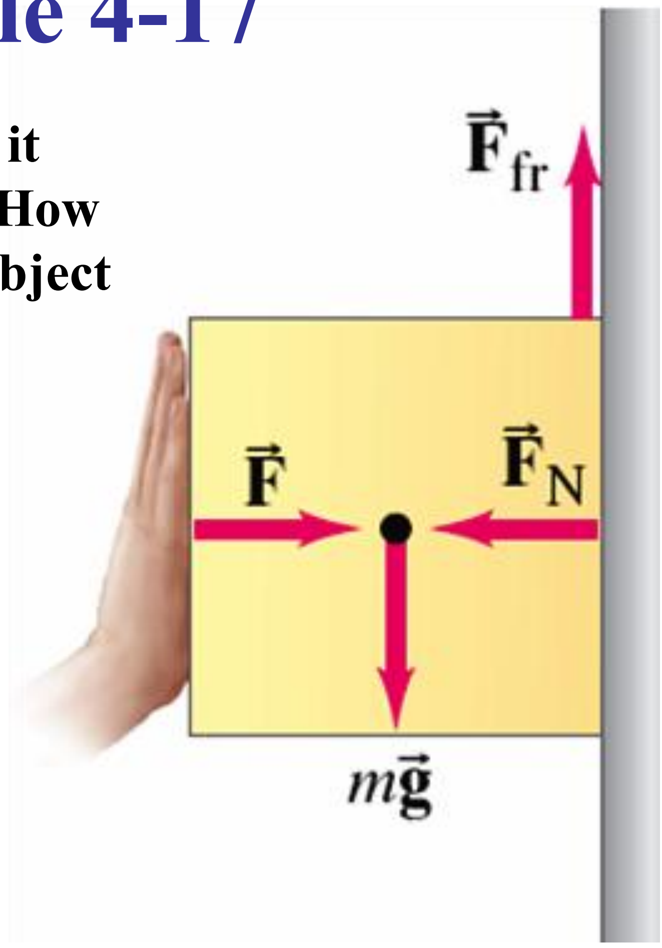
$$F_{fr} = \mu_k F_N = (0.30)(98.0\,\text{N}) = 29\,\text{N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\,\text{N} - 29\,\text{N} = 11\,\text{N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2$$

# Conceptual Example 4-17

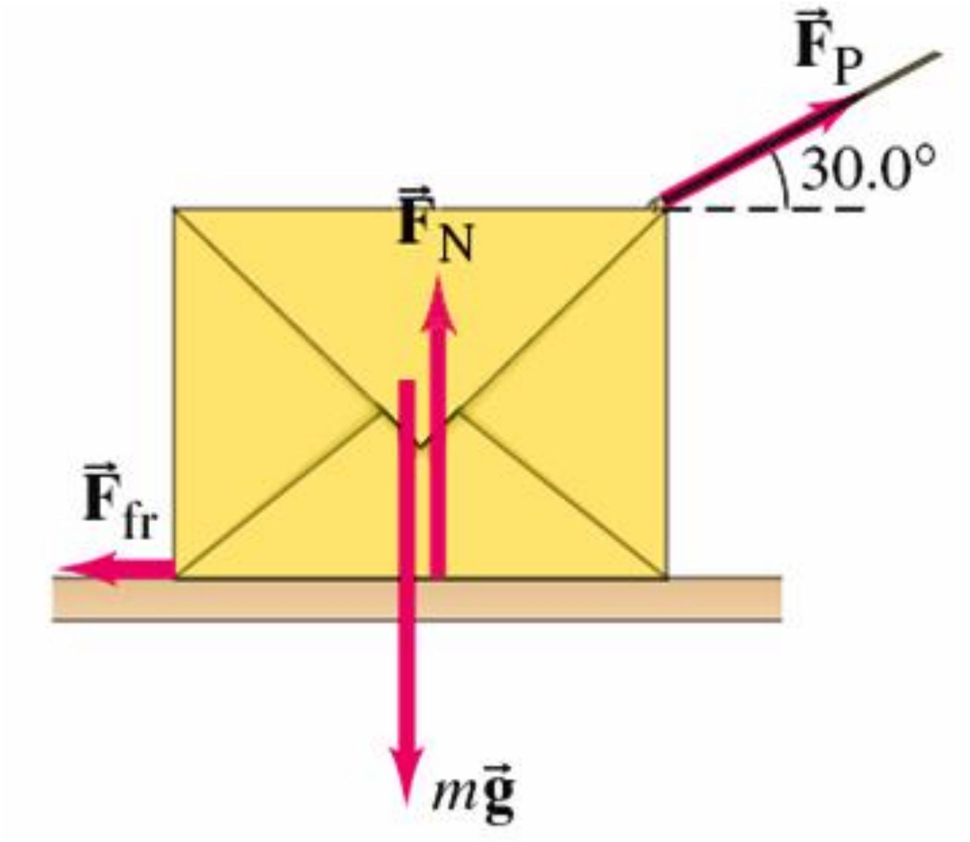
**You can hold a box against a rough wall & prevent it from slipping down by pressing hard horizontally. How does the application of a horizontal force keep an object from moving vertically?**




**RESPONSE** This won't work well if the wall is slippery. You need friction. Even then, if you don't press hard enough, the box will slip. The horizontal force you apply produces a normal force on the box exerted by the wall (the net force horizontally is zero since the box doesn't move horizontally). The force of gravity $mg$, acting downward on the box, can now be balanced by an upward static friction force whose maximum magnitude is proportional to the normal force. The harder you push, the greater $F_N$ is and the greater $F_{fr}$ can be. If you don't press hard enough, then $mg > \mu_s F_N$ and the box begins to slide down.

# Example 4-18: Pulling against friction

**A box, mass m = 10 kg, is pulled along a horizontal surface by a force of $F_P$ = 40.0 N applied at a 30.0° angle above horizontal. The coefficient of kinetic friction is $\mu_k$ = 0.3. Calculate the acceleration.**



# Example 4-18: Pulling against friction

**Solution:**

**Newton's 2nd Law in the y axis**

$$F_N - mg + F_{Py} = ma_y$$

$$F_N - 98.0\,\text{N} + 20.0\,\text{N} = 0,$$

$$F_N = 78.0\,\text{N}.$$

**Newton's 2nd Law in the x axis**

$$F_{Px} - F_{fr} = ma_x.$$

$$F_{fr} = \mu_k F_N = (0.30)(78.0\,\text{N}) = 23.4\,\text{N}.$$

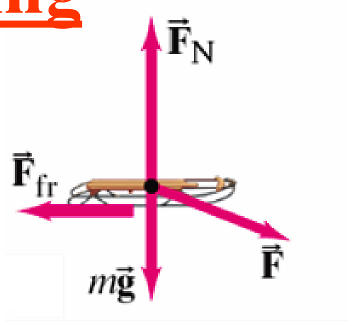
$$a_x = \frac{F_{Px} - F_{fr}}{m} = \frac{34.6\,\text{N} - 23.4\,\text{N}}{10.0\,\text{kg}} = 1.1\,\text{m/s}^2.$$

# Conceptual Example 4-19: To push or to pull a sled?

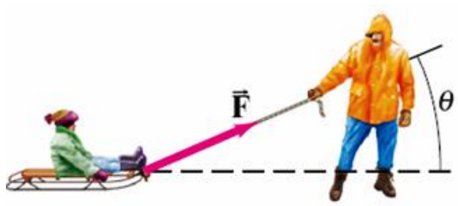
**Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? Assume the same angle θ in each case.**

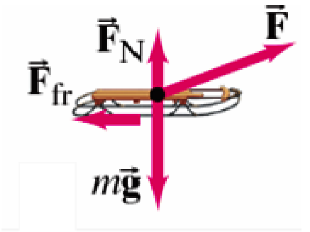
$$\sum F = ma$$

## Pushing


## Pulling




**Pushing**

y forces: $\sum F_y = 0$

$F_N - mg - F_y = 0$

$F_N = mg + F_y$

$F_{fr}(max) = \mu_s F_N$

x forces: $\sum F_x = ma$

**Pulling**

y forces: $\sum F_y = 0$

$F_N - mg + F_y = 0$

$F_N = mg - F_y$

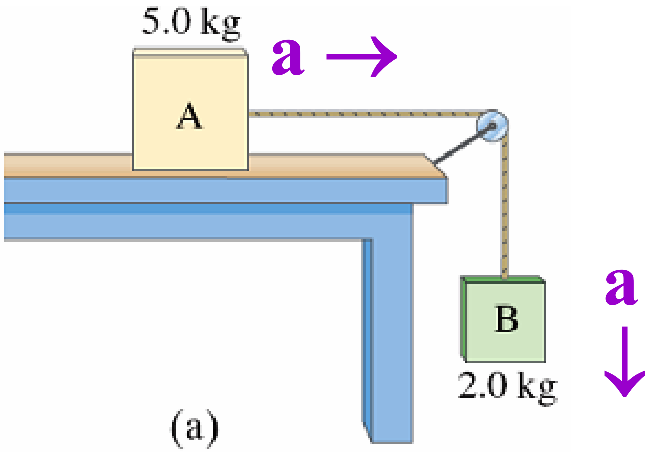
$F_{fr}(max) = \mu_s F_N$

x forces: $\sum F_x = ma$

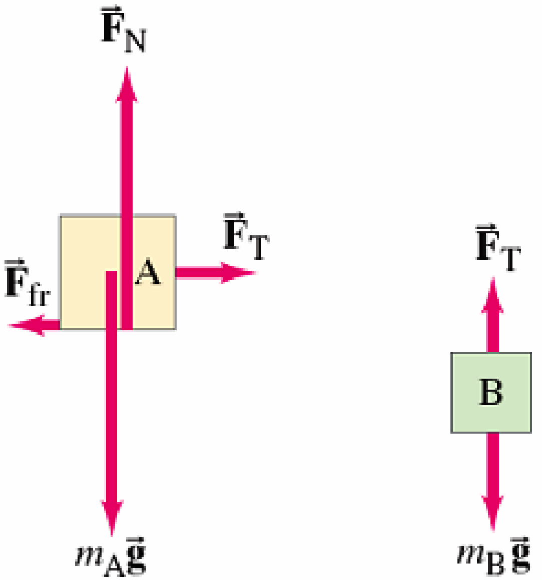
Because the friction force is proportional to the normal force, $F_{fr}$ will be less if you pull her. So you exert less force if you pull her.

# Example 4-20: Two boxes and a pulley

2 boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box A & the table is 0.2. Ignore mass of cord & pulley & friction in the pulley, which means a force applied to one end of the cord has the same magnitude at the other end. Find the acceleration, a, of the system, which has the same magnitude for both boxes if the cord doesn't stretch. As box B moves down, box A moves to the right.



(a)




$$\sum F = ma$$

***For <u>EACH</u> mass separately!***

**x & y components**

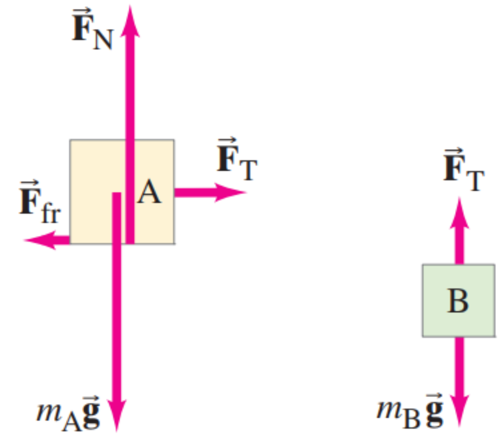
***plus*** $F_{fr} = \mu_k F_N$

# Example 4-20: Two boxes and a pulley

## Solution:

$$F_N = m_A g = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N}.$$

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$




$$\Sigma F_{Ax} = F_T - F_{fr} = m_A a. \qquad \text{[box A]}$$

$$\Sigma F_{By} = m_B g - F_T = m_B a. \qquad \text{[box B]}$$

We have two unknowns, $a$ and $F_T$, and we also have two equations. We solve the box A equation for $F_T$:

$$F_T = F_{fr} + m_A a,$$

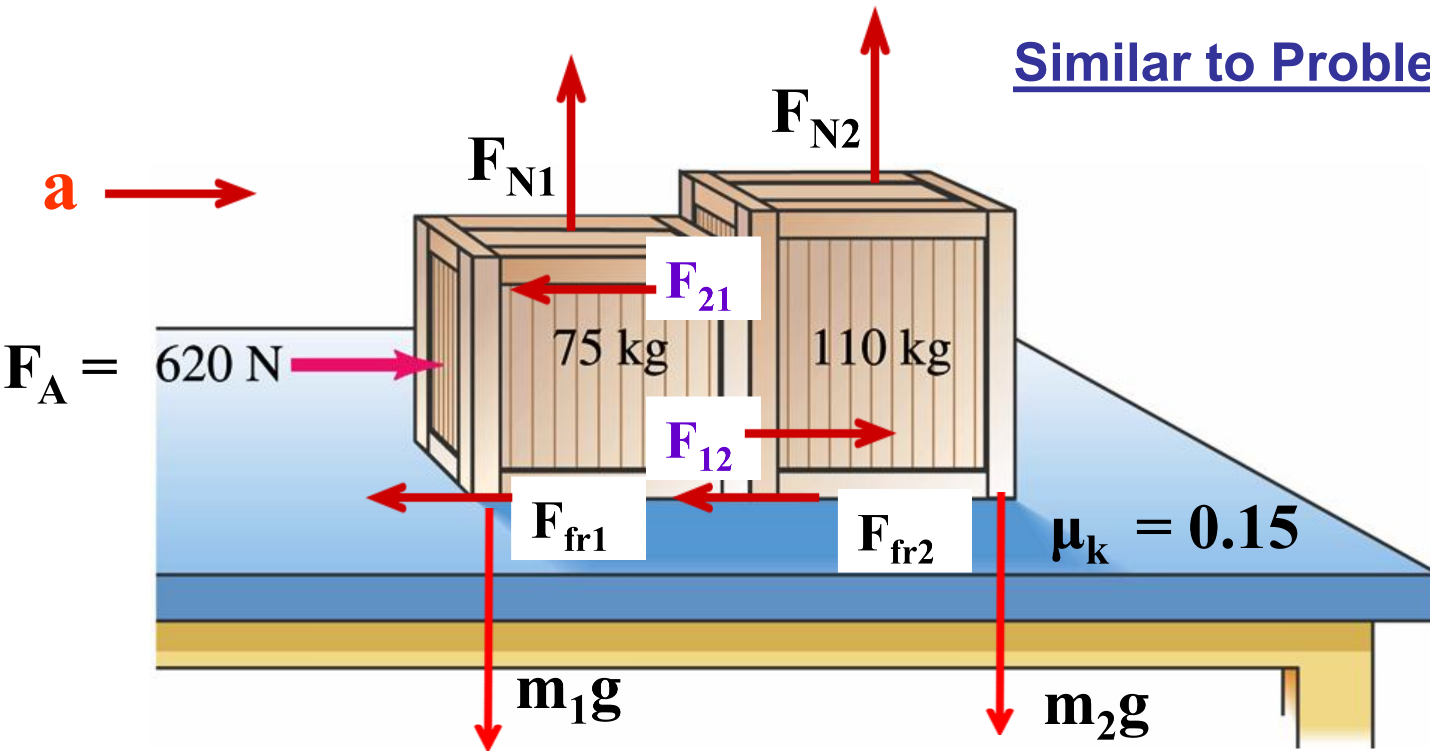
and substitute this into the box B equation:

$$m_B g - F_{fr} - m_A a = m_B a.$$

Now we solve for $a$ and put in numerical values:

$$a = \frac{m_B g - F_{fr}}{m_A + m_B} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

## Similar to Problem 47




$m_1 = 75$ kg
$m_2 = 110$ kg

**$m_1$:** $\sum F = m_1 a;$ x: $F_A - F_{21} - F_{fr1} = m_1 a$
y: $F_{N1} - m_1 g = 0$

**$m_2$:** $\sum F = m_1 a;$ x: $F_{12} - F_{fr2} = m_2 a$
y: $F_{N2} - m_2 g = 0$

**Friction:** $F_{fr1} = \mu_k F_{N1}; \; F_{fr2} = \mu_k F_{N2}$

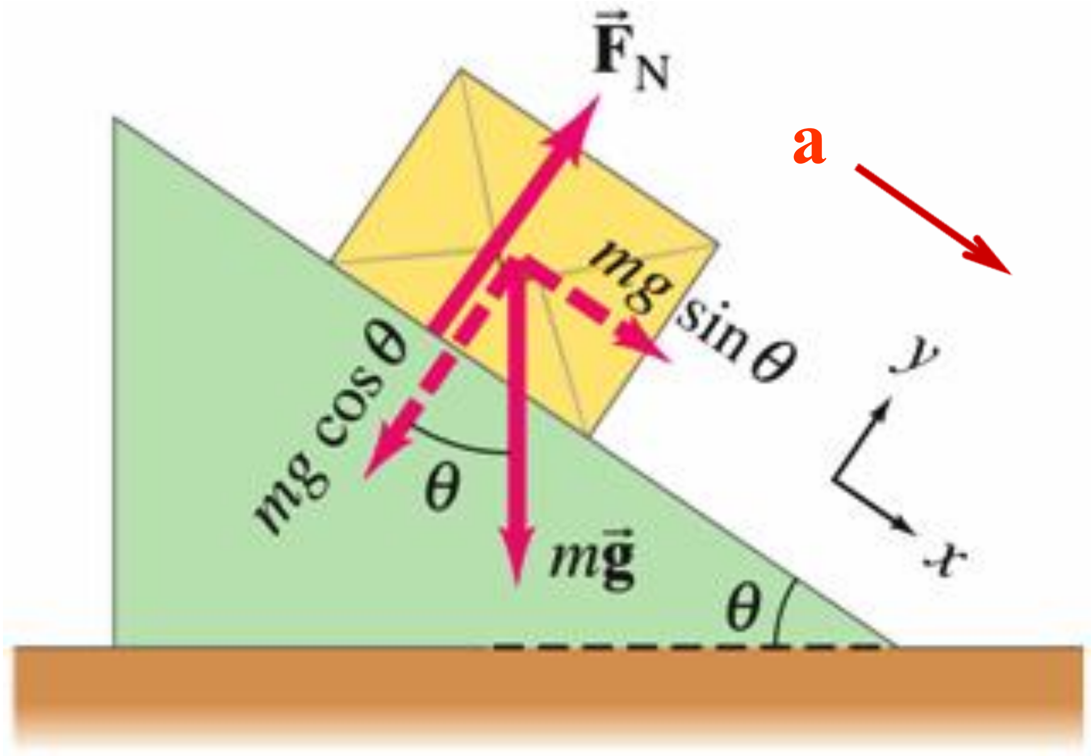
**3rd Law:** $F_{21} = - F_{12}$

$a = 1.88 \text{ m/s}^2$
$F_{12} = 368.5$ N

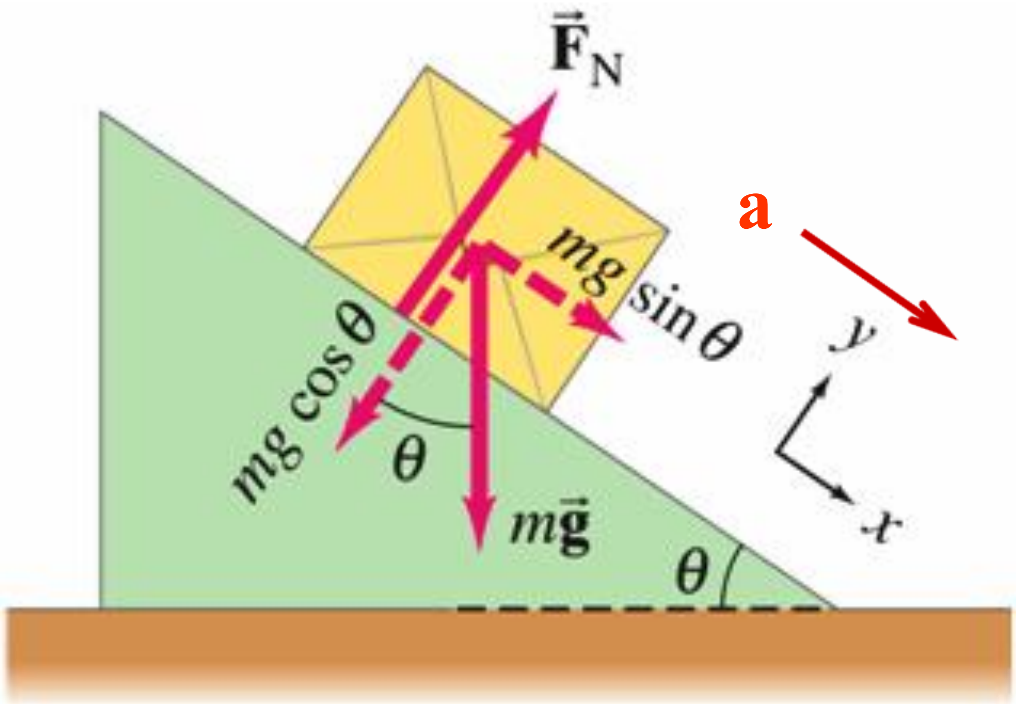
# Inclined Plane Problems

A tilted coordinate system. **MUST understand this!**

**Convenient, but not necessary.**




**Understand** $\sum$**F = ma** & how to resolve it into **x,y** components in the tilted coordinate system!!




You ***MUST*** understand this case to understand the case with friction!! By geometry, the 2 angles marked θ are the same!

$$F_G = mg$$

## *By Trigonometry:*

$$F_{Gx} = F_G \sin(\theta) = mg\sin(\theta)$$

$$F_{Gy} = -F_G \cos(\theta) = -mg\cos(\theta)$$

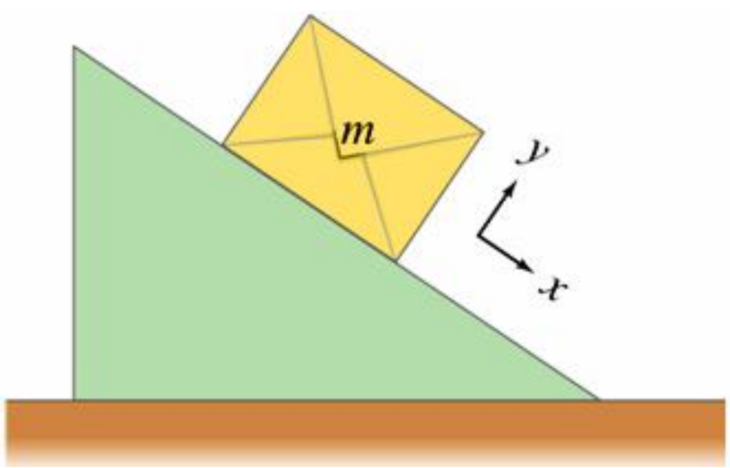
# Example: Sliding Down Incline

**A box of mass m is placed on a smooth (frictionless!) incline that makes an angle θ with the horizontal. Calculate:**

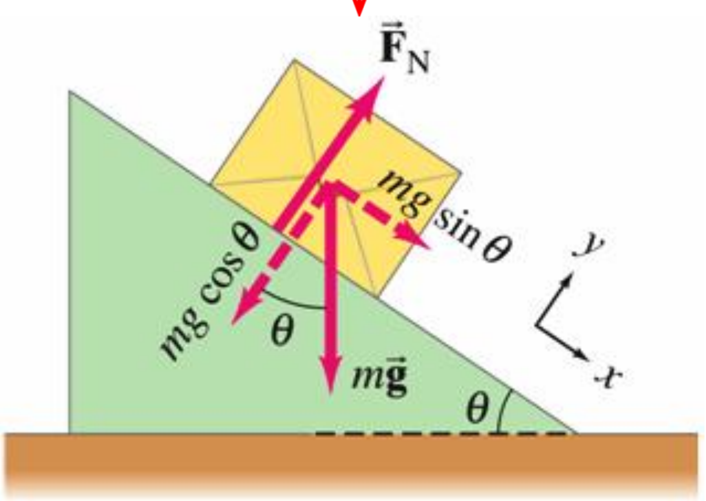
**a. The normal force on the box.**

**b. The box's acceleration.**

**c. Evaluate both for m = 10 kg & θ = 30º**

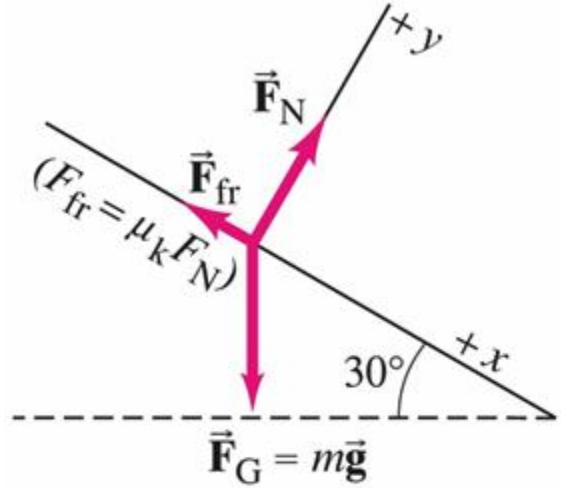


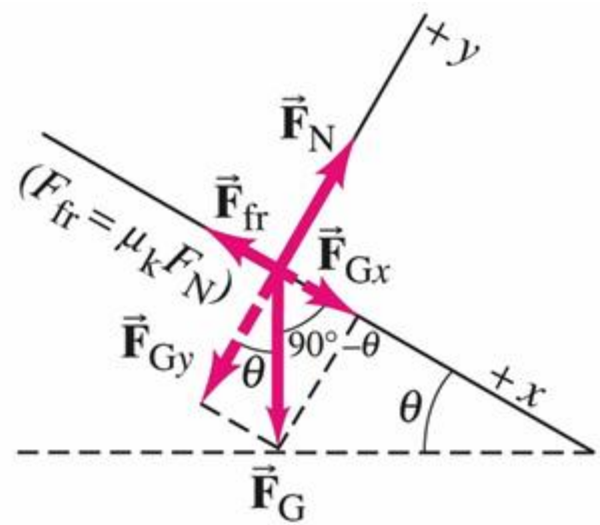

**Free Body Diagram**



# Example 4-21: The skier

**This skier is descending a 30° slope, at constant speed. What can you say about the coefficient of kinetic friction?**

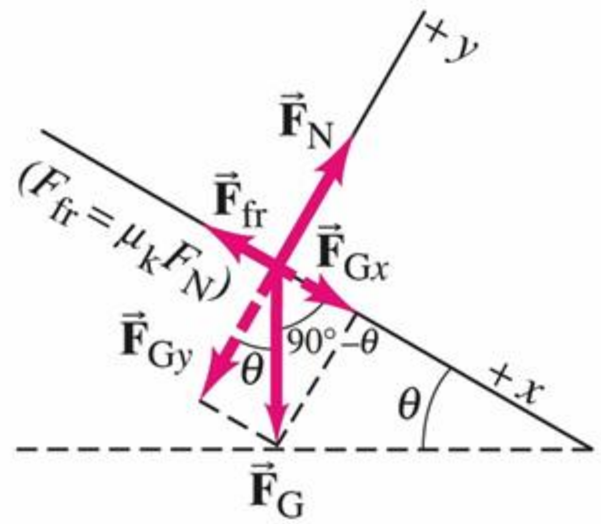

**Is the normal force $F_N$ equal & opposite to the weight???**

**NO!!!!!!!**

# Example 4-21: The skier

**EXAMPLE 4–21** **The skier.** The skier in Fig. 4–34a has begun descending the 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration?

**Solution:**




$$\Sigma F_y = ma_y$$

$$F_N - mg\cos\theta = ma_y = 0$$

$$F_N = mg\cos\theta$$

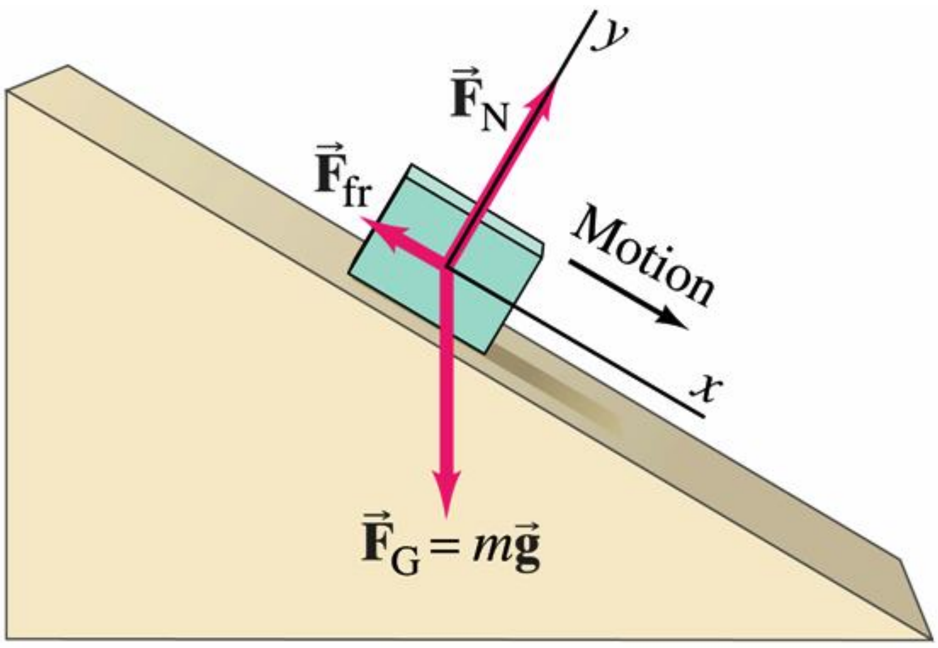
$$\Sigma F_x = ma_x$$

$$mg\sin\theta - \mu_k F_N = ma_x$$

$$a_x = g\sin 30^\circ - \mu_k g\cos 30^\circ$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

$$a = (0.41)(9.8\,\text{m/s}^2) = 4.0\,\text{m/s}^2.$$

**Summary of Inclines**: An object sliding down an incline has ***3 forces acting on it***: the normal force $F_N$, gravity $F_G$ = mg, & friction $F_{fr}$. The normal force $F_N$ is always perpendicular to the surface & is ***NOT*** equal & opposite to the weight mg. The friction force $F_{fr}$ is parallel to the surface. Gravity (weight) $F_G$ = mg points down.




If the object is at rest, the forces are the same except that we use the static frictional force, and the sum of the forces is zero.

# Problem 52

$\sum F = ma$

x: $mg\sin\theta - F_f = ma$

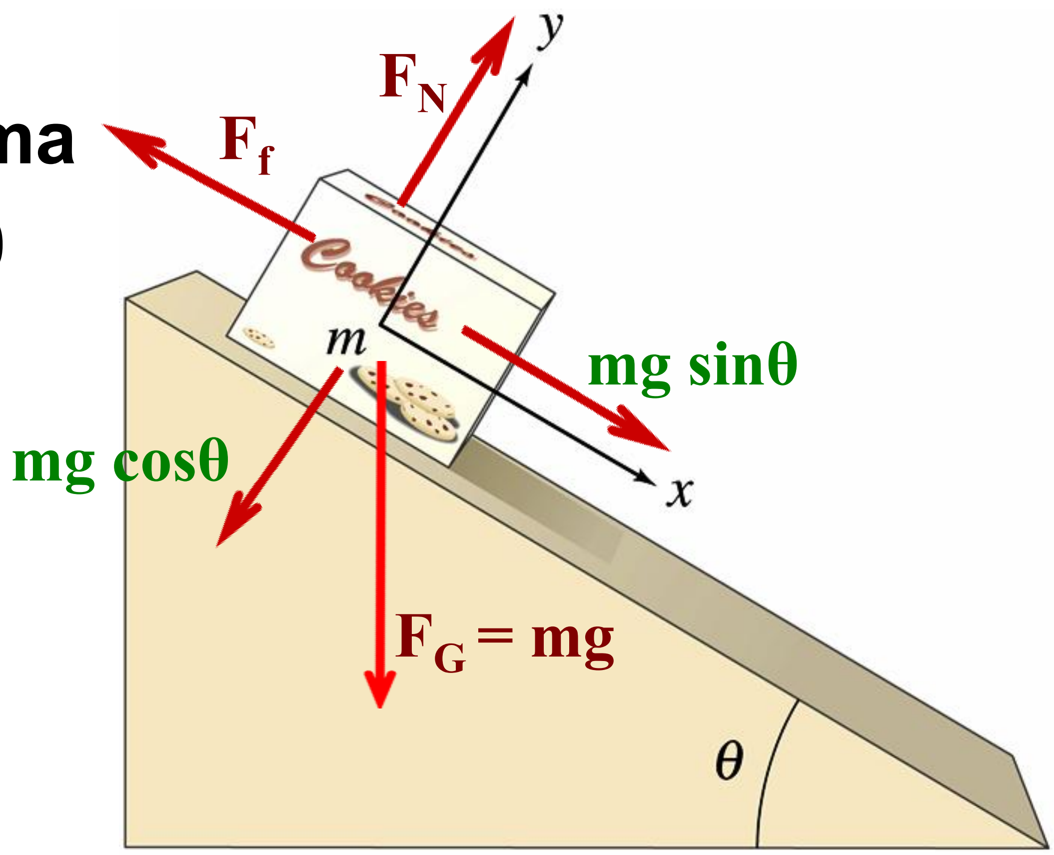
y: $F_N - mg\cos\theta = 0$

Friction: $F_f = \mu_k F_N$

***NOTE!!!***

$\Rightarrow F_N = mg\cos\theta$

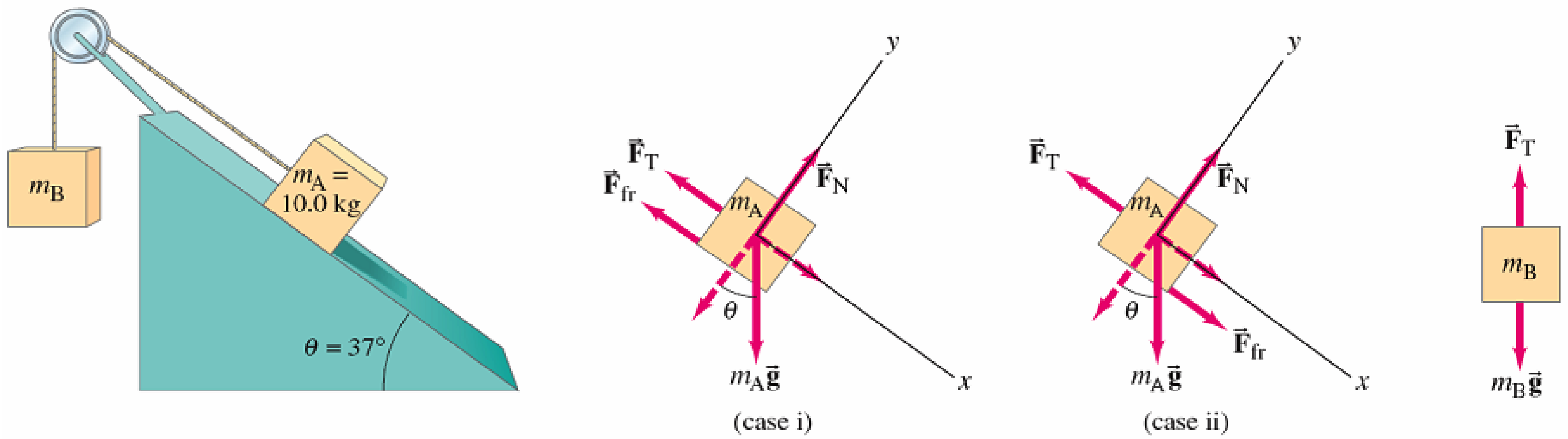
$\neq mg$




***THE NORMAL FORCE IS NOT EQUAL TO THE WEIGHT!!!***

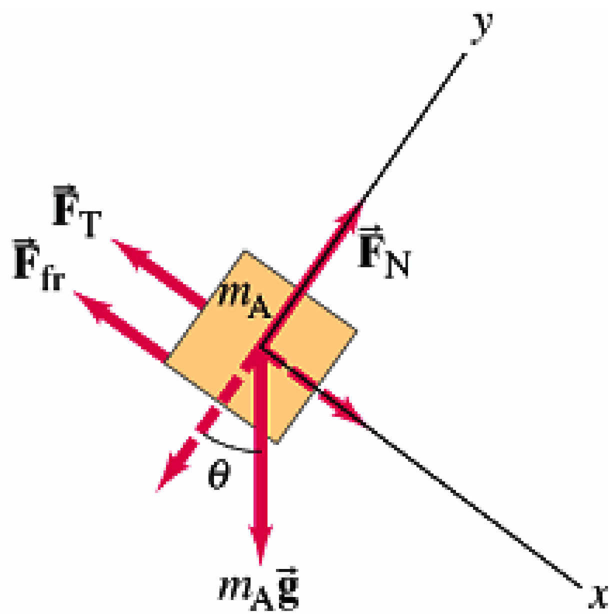
# Example: A ramp, a pulley, & two boxes

**Box A, mass $m_A$ = 10 kg, rests on a surface inclined at $\theta$ = 37° to the horizontal. It's connected by a light cord, passing over a massless, frictionless pulley, to Box B, which hangs freely. (a) If the coefficient of static friction is $\mu_s$ = 0.4, find the range of values for mass B which will keep the system at rest. (b) If the coefficient of kinetic friction is $\mu_k$ = 0.3, and $m_B$ = 10 kg, find the acceleration of the system.**

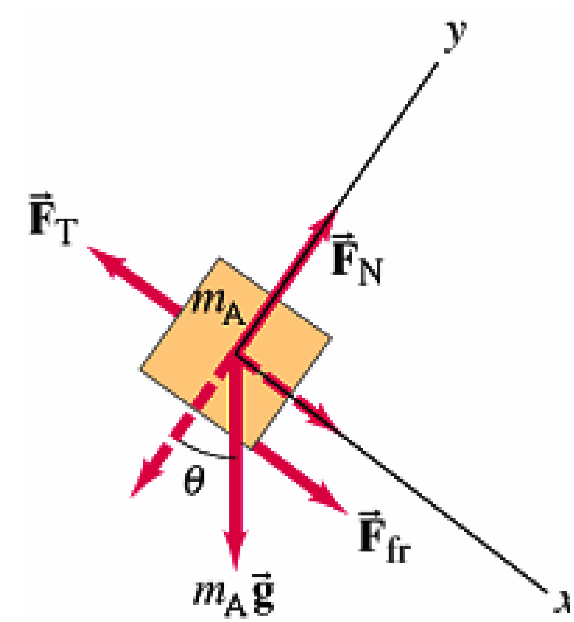


# Example Continued

**(a) Coefficient of static friction $\mu_s = 0.4$, find the range of values for mass B which will keep the system at rest. Static Case (i): Small $m_B << m_A$: $m_A$ slides down the incline, so friction acts up the incline. Static Case (ii): Larger $m_B > m_A$: $m_A$ slides up the incline, so friction acts down the incline.**



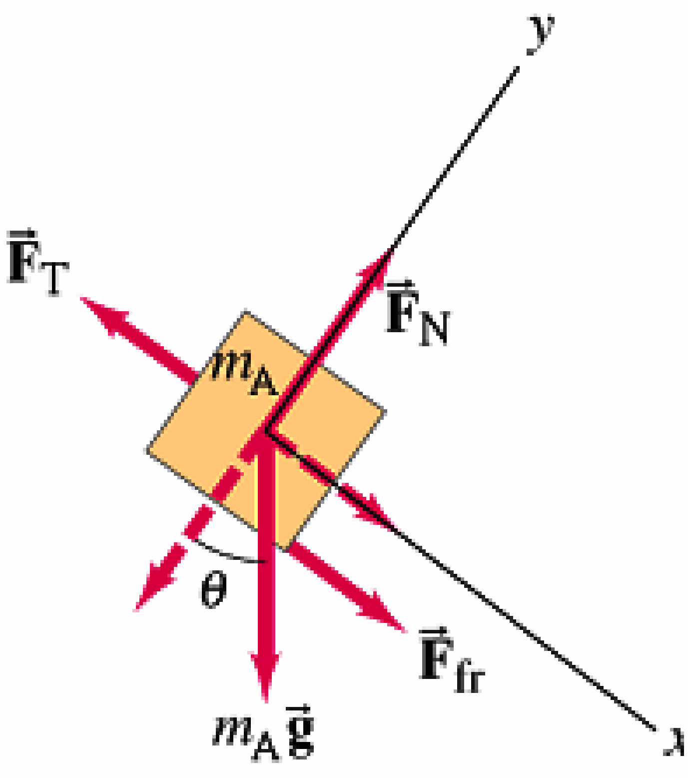

**(i): $m_B << m_A$**
**$m_A$ slides down incline**
**$F_{fr}$ acts up incline**




**(ii): Larger $m_B > m_A$**
**$m_A$ slides up incline**
**$F_{fr}$ acts down incline**

# Example Continued

**(b) If the coefficient of kinetic friction is $\mu_k = 0.3$, and $m_B = 10$ kg, find the acceleration of the system & the tension in the cord.**




**Motion: $m_B = 10$ kg**

**$m_A$ slides up incline**

**$F_{fr}$ acts down incline**

# Problem 3

(I) How much tension must a rope withstand if it is used to accelerate a 1210-kg car horizontally along a frictionless surface at $1.20\ \mathrm{m/s^2}$?

# Solution:

Use Newton's second law to calculate the tension.

$$\sum F = F_{\mathrm{T}} = ma = (1210\ \mathrm{kg})(1.20\ \mathrm{m/s^2}) = 1452\ \mathrm{N} \approx \boxed{1450\ \mathrm{N}}$$

# Problem 11

(II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 4–43. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.

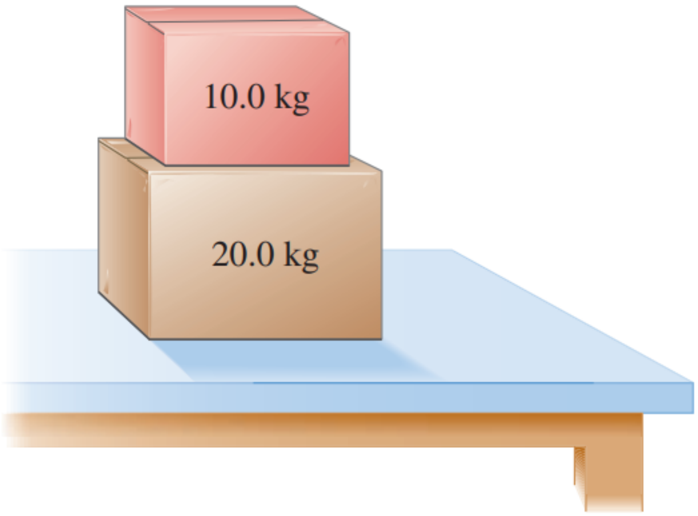
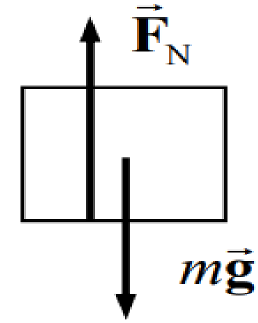


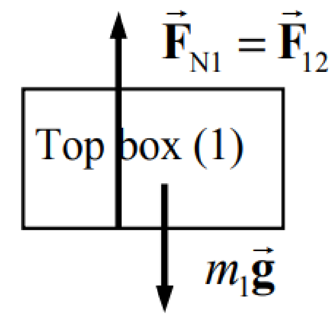

**FIGURE 4–43**
Problem 11.

# Solution:

(*a*) The 20.0-kg box resting on the table has the free-body diagram shown. Its weight is $mg = (20.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{196\text{ N}}$. Since the box is at rest, the net force on the box must be 0, so the normal force must also be $\boxed{196\text{ N}}$.




(*b*) Free-body diagrams are shown for both boxes. $\vec{\mathbf{F}}_{12}$ is the force on box 1 (the top box) due to box 2 (the bottom box), and is the normal force on box 1. $\vec{\mathbf{F}}_{21}$ is the force on box 2 due to box 1, and has the same magnitude as $\vec{\mathbf{F}}_{12}$ by Newton's third law. $\vec{\mathbf{F}}_{\text{N}2}$ is the force of the table on box 2. That is

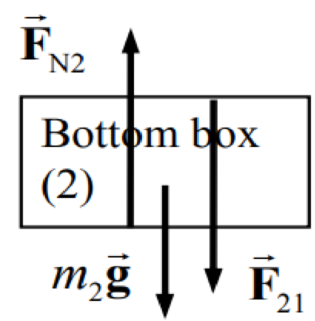



$$\sum F_1 = F_{\text{N1}} - m_1 g = 0$$

$$F_{\text{N1}} = m_1 g = (10.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{98.0\text{ N}} = F_{12} = F_{21}$$

$$\sum F_2 = F_{\text{N2}} - F_{21} - m_2 g = 0$$

$$F_{\text{N2}} = F_{21} + m_2 g = 98.0\text{ N} + (20.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{294\text{ N}}$$



## Problem 28

(II) The two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ shown in Fig. 4–52a and b (looking down) act on an 18.5-kg object on a frictionless tabletop. If $F_1 = 10.2\,\mathrm{N}$ and $F_2 = 16.0\,\mathrm{N}$, find the net force on the object and its acceleration for (a) and (b).

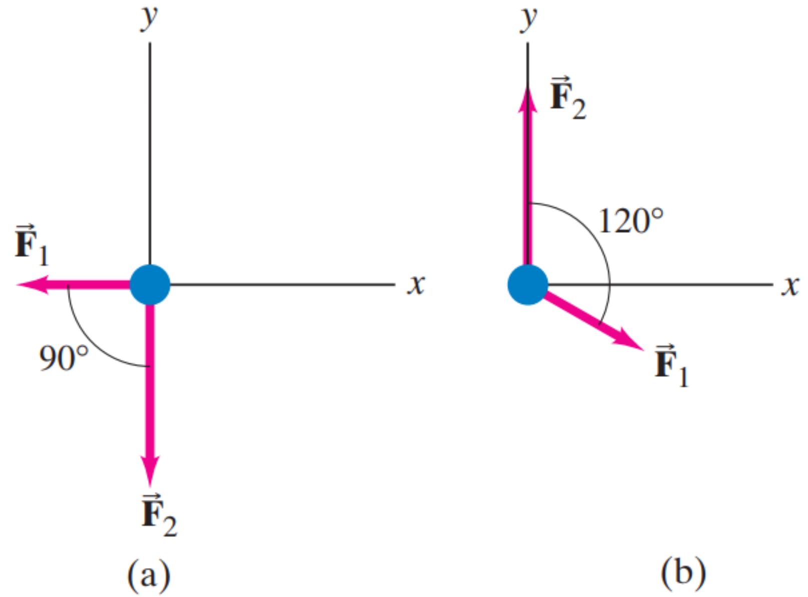



**FIGURE 4–52** Problem 28.

# Solution:

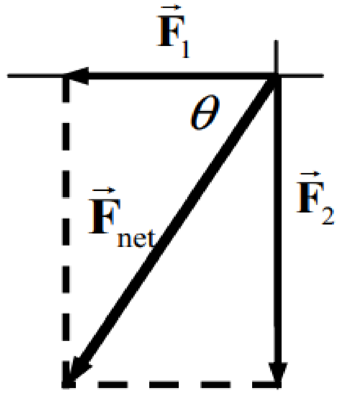
(a) $F_{\text{net}\,x} = -F_1 = -10.2\text{ N} \qquad F_{\text{net}\,y} = -F_2 = -16.0\text{ N}$

$F_{\text{net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = 19.0\text{ N} \qquad \theta = \tan^{-1}\dfrac{-16.0}{-10.2} = 57.48°$

The actual angle from the $x$ axis is then $237.48°$. Thus the net force is

$F_{\text{net}} = \boxed{19.0\text{ N at }237°}$

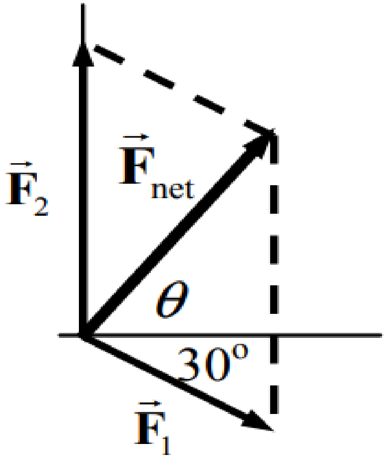
$a = \dfrac{F_{\text{net}}}{m} = \dfrac{19.0\text{ N}}{18.5\text{ kg}} = \boxed{1.03\text{ m/s}^2\text{ at }237°}$




(b) $F_{\text{net}\,x} = F_1\cos 30^\circ = 8.833\text{ N} \qquad F_{\text{net}\,y} = F_2 - F_1\sin 30^\circ = 10.9\text{ N}$

$F_{\text{net}} = \sqrt{(8.833\text{ N})^2 + (10.9\text{ N})^2} = 14.03\text{N} \approx \boxed{14.0\text{ N}}$

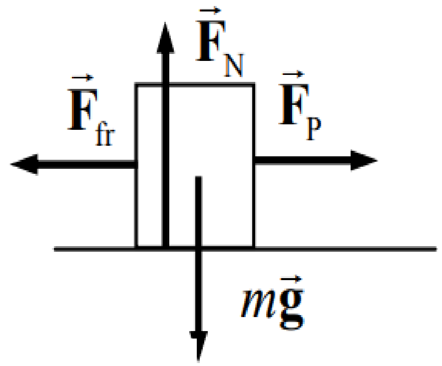
$\theta = \tan^{-1}\dfrac{10.9}{8.833} = \boxed{51.0°} \qquad a = \dfrac{F_{\text{net}}}{m} = \dfrac{14.03\text{ N}}{18.5\text{ kg}} = \boxed{0.758\text{ m/s}^2\text{ at }51.0°}$



# Problem 36

(I) A force of 35.0 N is required to start a 6.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 35.0-N force continues, the box accelerates at $0.60\ m/s^2$. What is the coefficient of kinetic friction?

# Solution:




A free-body diagram for the box is shown. Since the box does not accelerate vertically, $F_N = mg$.

(*a*) To start the box moving, the pulling force must just overcome the force of static friction, and that means the force of static friction will reach its maximum value of $F_{fr} = \mu_s F_N$. Thus, we have for the starting motion,

$$\sum F_x = F_P - F_{fr} = 0 \quad \rightarrow$$

$$F_P = F_{fr} = \mu_s F_N = \mu_s mg \quad \rightarrow \quad \mu_s = \frac{F_P}{mg} = \frac{35.0\text{ N}}{(6.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.60}$$

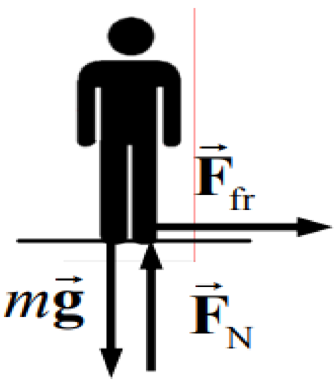
(*b*) The same force diagram applies, but now the friction is kinetic friction, and the pulling force is NOT equal to the frictional force, since the box is accelerating to the right.

$$\sum F = F_P - F_{fr} = ma \quad \rightarrow \quad F_P - \mu_k F_N = ma \quad \rightarrow \quad F_P - \mu_k mg = ma \quad \rightarrow$$

$$\mu_k = \frac{F_P - ma}{mg} = \frac{35.0\text{ N} - (6.0\text{ kg})(0.60\text{ m/s}^2)}{(6.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.53}$$

# Problem 37

(I) Suppose you are standing on a train accelerating at 0.20 $g$. What minimum coefficient of static friction must exist between your feet and the floor if you are not to slide?




## Solution:

A free-body diagram for you as you stand on the train is shown. You do not accelerate vertically, so $F_{\text{N}} = mg$. The maximum static frictional force is $\mu_{\text{s}} F_{\text{N}}$, and that must be greater than or equal to the force needed to accelerate you in order for you not to slip.

$$F_{\text{fr}} \geq ma \quad \rightarrow \quad \mu_{\text{s}} F_{\text{N}} \geq ma \quad \rightarrow \quad \mu_{\text{s}} mg \geq ma \quad \rightarrow \quad \mu_{\text{s}} \geq a/g = 0.20g/g = \boxed{0.20}$$

The static coefficient of friction must be at least 0.20 for you not to slide.

## Problem 45

(II) In Fig. 4–56 the coefficient of static friction between mass $m_A$ and the table is 0.40, whereas the coefficient of kinetic friction is 0.20. (*a*) What minimum value of $m_A$ will keep the system from starting to move? (*b*) What value(s) of $m_A$ will keep the system moving at constant speed? [Ignore masses of the cord and the (frictionless) pulley.]

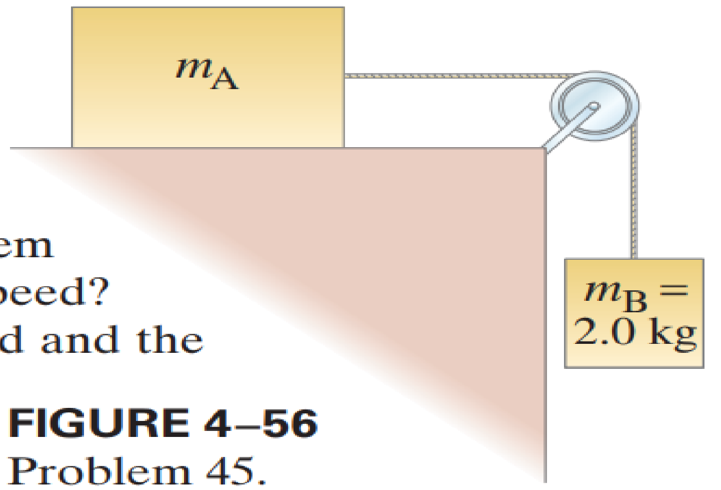



**FIGURE 4–56**
Problem 45.

## Solution:

(*a*) For $m_B$ not to move, the tension must be equal to $m_B g$, so $m_B g = F_T$. For $m_A$ not to move, the tension must be equal to the force of static friction, so $F_{fr} = F_T$. Note that the normal force on $m_A$ is equal to its weight. Use these relationships to solve for $m_A$.

$$m_B g = F_T = F_{fr} \le \mu_s m_A g \quad \rightarrow \quad m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\text{ kg}}{0.40} = 5.0\text{ kg} \quad \rightarrow \quad m_A \ge \boxed{5.0\text{ kg}}$$

(*b*) For $m_B$ to move with constant velocity, the tension must be equal to $m_B g$. For $m_A$ to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on $m_A$ is equal to its weight. Use these relationships to solve for $m_A$.

$$m_B g = F_{fr} = \mu_k m_A g \quad \rightarrow \quad m_A = \frac{m_B}{\mu_k} = \frac{2.0\text{ kg}}{0.20} = \boxed{10\text{ kg}}\text{(2 significant figures)}$$