Physics 105

CH30: Nuclear Physics and Radioactivity

Department of Physics
The University of Jordan

30-1 Structure and Properties of the Nucleus

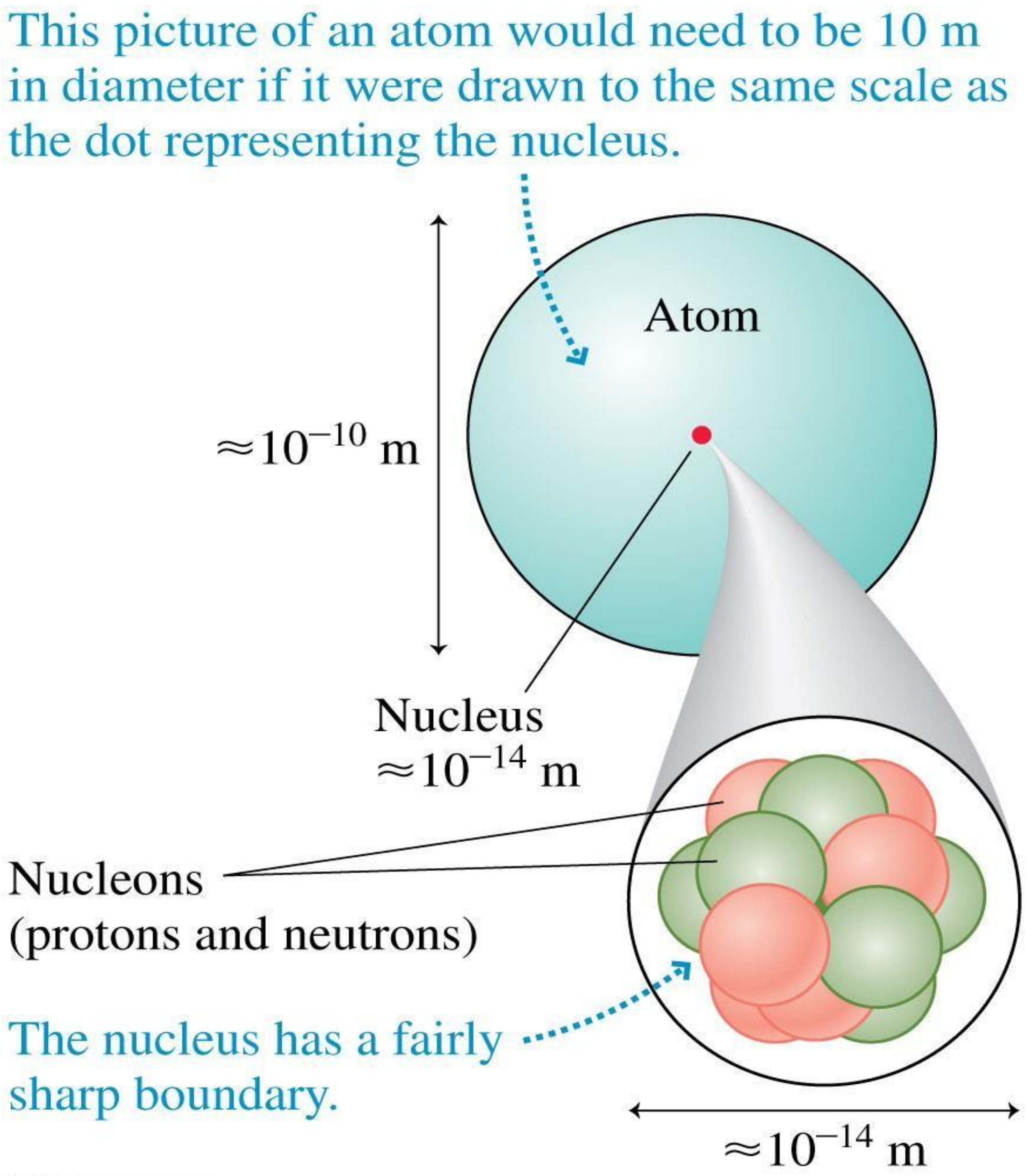
- The nucleus is composed of protons and neutrons. Together they are referred to as nucleons.

Nucleus is made of protons and neutrons Proton has positive charge; here is its mass:

$m_p = 1.67262 \times 10^{-27}$ kg

Neutron is electrically neutral, and slightly more massive than the proton:

$m_n = 1.67493 \times 10^{-27}$ kg



Neutrons and protons are collectively called nucleons.

Number of protons: atomic number, Z Number of

nucleons: atomic mass number, A

Neutron number: $N = A - Z$

A and Z are sufficient to specify a nuclide. Nuclides are symbolized as follows:

$$^{A}_{Z}\mathrm{X},$$

X is the chemical symbol for the element; it contains the same information as Z but in a more easily recognizable form.

Isotopes

The leading superscript gives the total number of nucleons, which is the mass number *A*.

$^{12}_{6}C$ $^{13}_{6}C$ $^{14}_{6}C$

The leading subscript (if included) gives the number of protons.

The three nuclei all have the same number of protons, so they are isotopes of the same element, carbon.

- Nuclei with the same Z—so they are the same element—but different N are called isotopes. For many elements, several different isotopes exist in nature.
- Because of wave-particle duality, the size of the nucleus is somewhat fuzzy.
- It is found that nuclei have a roughly spherical shape with a radius that increases with A according to the approximate formula:

$$r \approx \left(1.2 \times 10^{-15}\ \mathrm{m}\right)\left(A^{\frac{1}{3}}\right).$$

EXAMPLE 30–1 ESTIMATE **Nuclear sizes.** Estimate the diameter of the smallest and largest naturally occurring nuclei: (*a*) ${}^{1}_{1}H$, (*b*) ${}^{238}_{92}U$.

SOLUTION (*a*) For hydrogen, $A = 1$, Eq. 30–1 gives

$$d = \text{diameter} = 2r \approx 2\left(1.2 \times 10^{-15}\,\text{m}\right)\left(A^{\frac{1}{3}}\right) = 2.4 \times 10^{-15}\,\text{m}$$

since $A^{\frac{1}{3}} = 1^{\frac{1}{3}} = 1.$

(*b*) For uranium $d \approx \left(2.4 \times 10^{-15}\,\text{m}\right)(238)^{\frac{1}{3}} = 15 \times 10^{-15}\,\text{m}.$

The range of nuclear diameters is only from 2.4 fm to 15 fm.

Problem 2:

(I) What is the approximate radius of an α particle ($^4_2$He)?

Solution:

The α particle is a helium nucleus and has $A = 4$. Use Eq. 30–1.

$$r = (1.2\times10^{-15}\text{ m})A^{\frac{1}{3}} = (1.2\times10^{-15}\text{ m})(4)^{\frac{1}{3}} = \boxed{1.9\times10^{-15}\text{ m}} = 1.9\text{ fm}$$

Atomic Mass

- The atomic masses are specified in terms of the *atomic mass unit* u, defined such that the atomic mass of isotope ^{12}C is exactly 12 u.
- $1\ u = 1.6605 \times 10^{-27}$ kg
- The energy equivalent of 1 u of mass is

$$E_0 = (1.6605 \times 10^{-27}\,\text{kg})(2.9979 \times 10^8\,\text{m/s})^2$$

$$= 1.4924 \times 10^{-10}\,\text{J} = 931.49\,\text{MeV}$$

- To find the energy equivalent of any atom or particle whose mass is given in atomic mass units we can use

$$E_0\,(\text{in MeV}) = m(\text{in u}) \times (931.49\,\text{MeV/u})$$

Masses of atoms are measured with reference to the carbon-12 atom, which is assigned a mass of exactly 12u. A u is a unified atomic mass unit.

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}$$

From the following table, you can see that the electron is considerably less massive than a nucleon.

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

	Mass		
Object	**kg**	**u**	**MeV/c^2**
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Atomic Mass

- We can write 1 u in the following form as well:

$$1\ \text{u} = \frac{E_0}{c^2} = 931.49\left(\frac{\text{MeV}}{c^2}\right)$$

- MeV/c^2 are units of mass. The energy equivalent of 1 MeV/c^2 is 1 MeV.

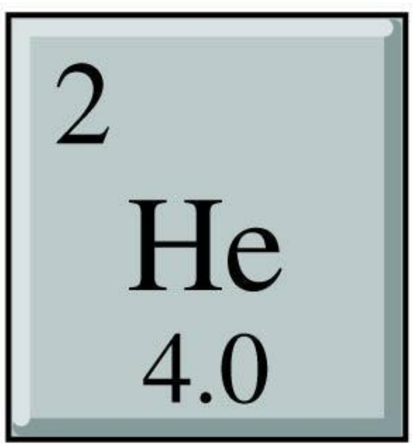
Atomic Mass

- The mass of a hydrogen atom is equal to the sum of the masses of a proton and an electron.
- The mass of a helium atom is *less* than the sum of the masses of its protons, neutrons, and electrons due to the *binding energy* of the nucleus.
- The *chemical* atomic mass shown on the periodic table is the *weighted average* of the atomic masses of all naturally occurring isotopes.

QuickCheck

The isotope ^{3}He has_____neutrons.

A. 0
B. 1
C. 2
D. 3
E. 4



QuickCheck

The isotope ^{3}He has_____neutrons.

He is the second element with 2 protons.

3 nucleons

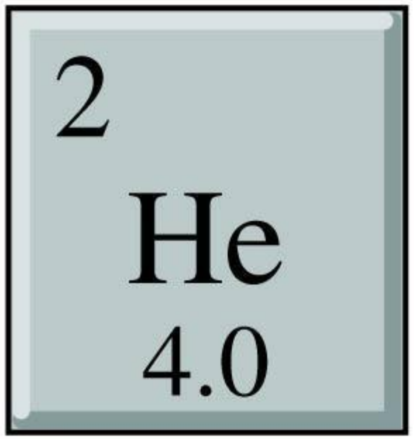
A. 0

B. 1

C. 2

D. 3

E. 4



30-3 Radioactivity

- Towards the end of the 19th century, minerals were found that would darken a photographic plate even in the absence of light.
- This phenomenon is now called radioactivity.
- Marie and Pierre Curie isolated two new elements that were highly radioactive; they are now called polonium and radium.
- Radioactivity is the result of the disintegration or decay of an unstable nucleus.

30-3 Radioactivity

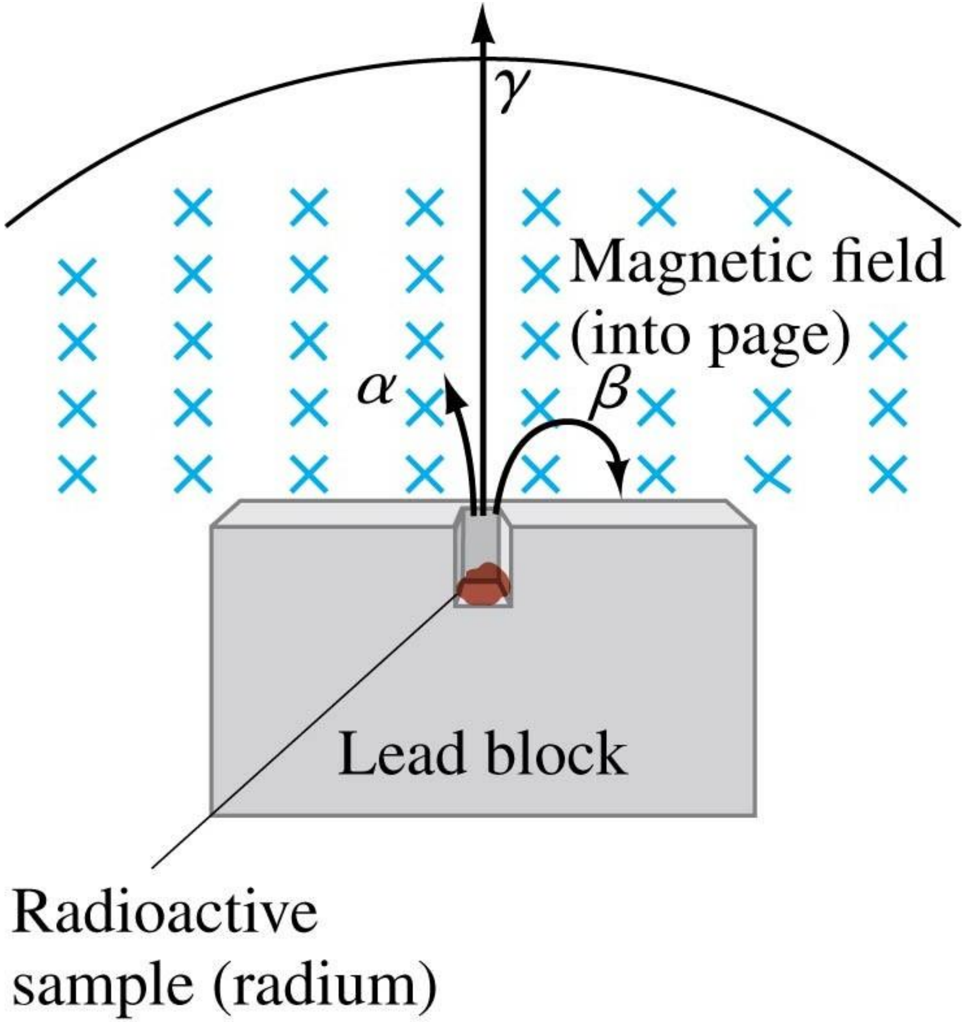
Radioactive rays were observed to be of three types:

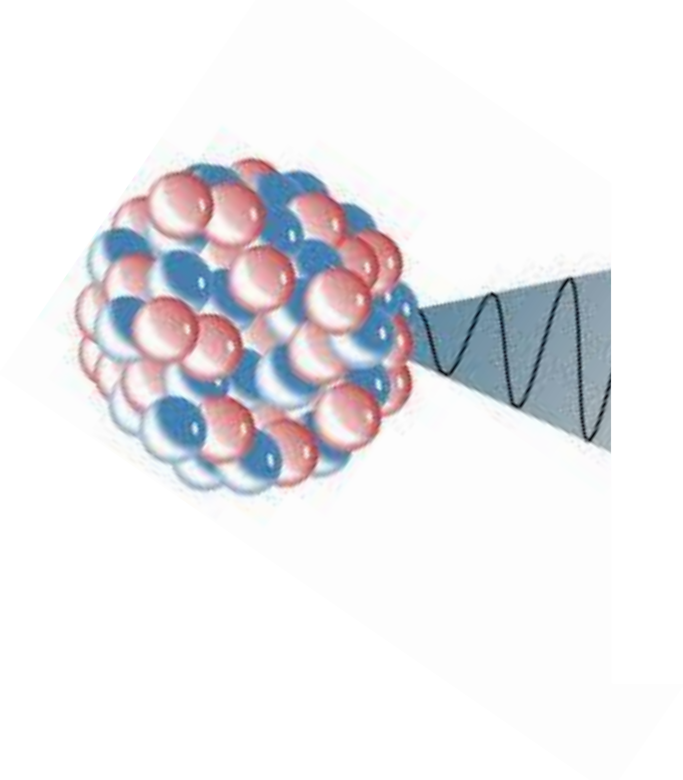
1. Alpha rays, which could barely penetrate a piece of paper
2. Beta rays, which could penetrate 3 mm of aluminum
3. Gamma rays, which could penetrate several centimeters of lead

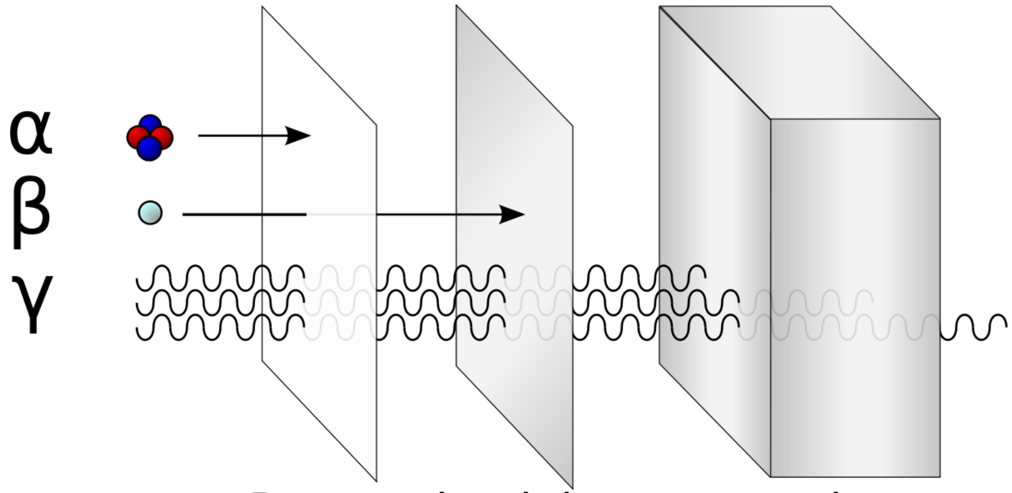
We now know that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are electromagnetic radiation.

30-3 Radioactivity

Alpha and beta rays are bent in opposite directions in a magnetic field, while gamma rays are not bent at all.



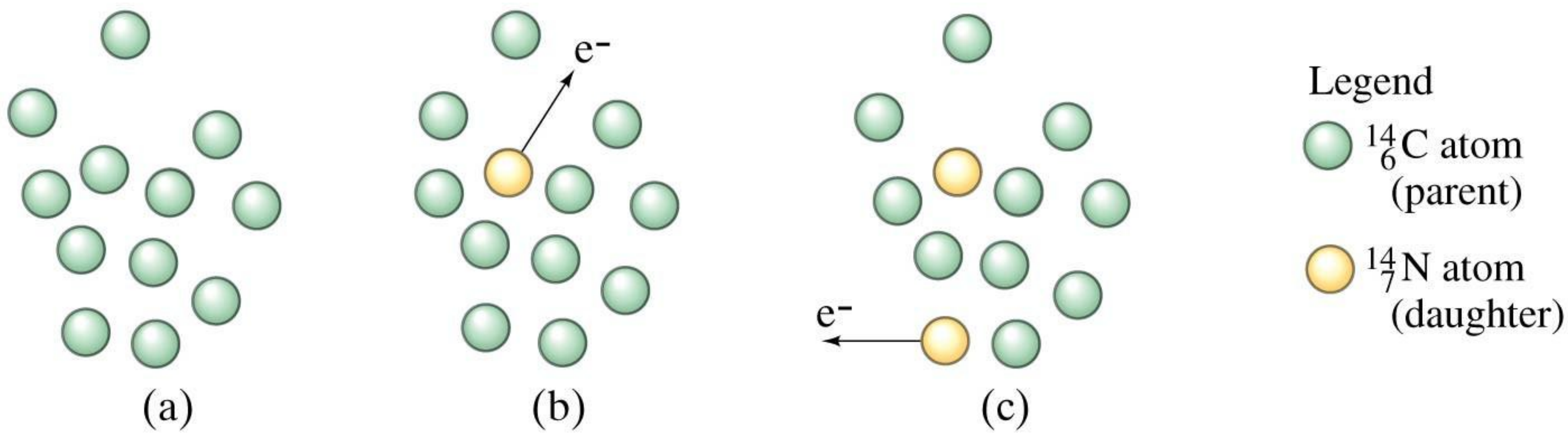




Particle	What is it	Charge	Range in air	Penetration	Ionisation
Alpha (α)	2 protons + 2 neutrons	+2	Few cm	Stopped by paper	High
Beta (β^-)	Electron	−1	Few 10s of cm	Stopped by few mm Aluminium	Medium
Gamma (γ)	Electromagnetic wave	0	Infinite	Reduced by few mm Lead	Low

30-8 Half-Life and Rate of Decay

Nuclear decay is a random process; the decay of any nucleus is not influenced by the decay of any other.



30-8 Half-Life and Rate of Decay

Therefore, the number of decays in a short time interval is proportional to the number of nuclei present and to the time:

$$\Delta N = -\lambda N \, \Delta t \quad (30\text{-}3a)$$

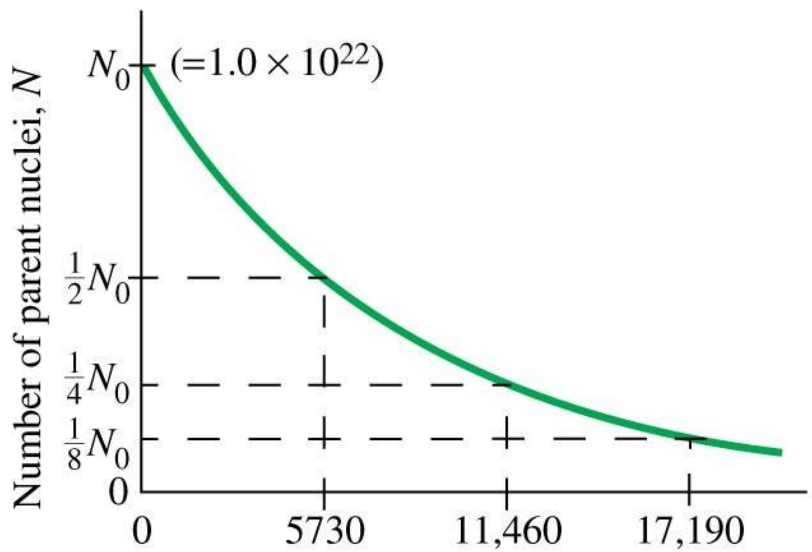
Here, λ is a constant characteristic of that particular nuclide, called the decay constant.

30-8 Half-Life and Rate of Decay

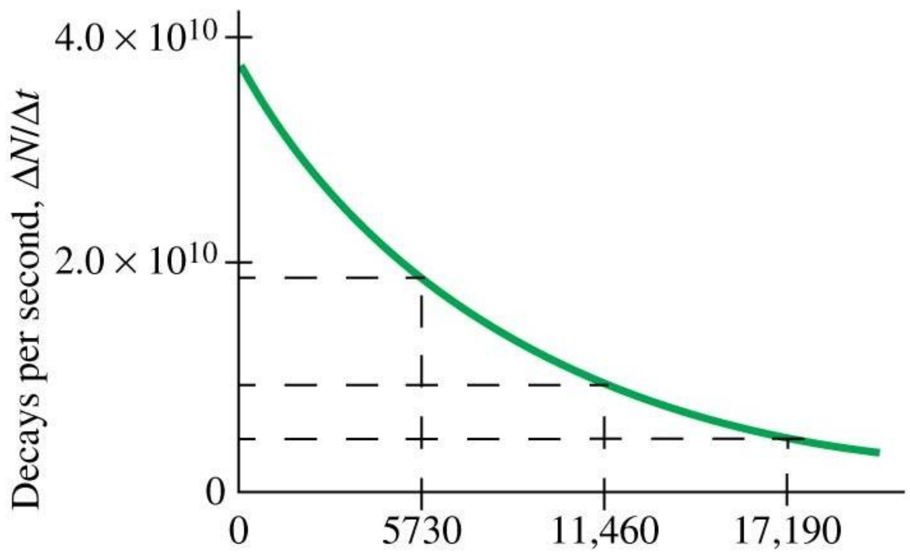
This equation can be solved, using calculus, for N as a function of time:

$$N = N_0 e^{-\lambda t}, \qquad (30\text{-}4)$$

Where N_o is the number of parent nuclei present at any chosen time t = 0 and N is the number remaining after a time t .



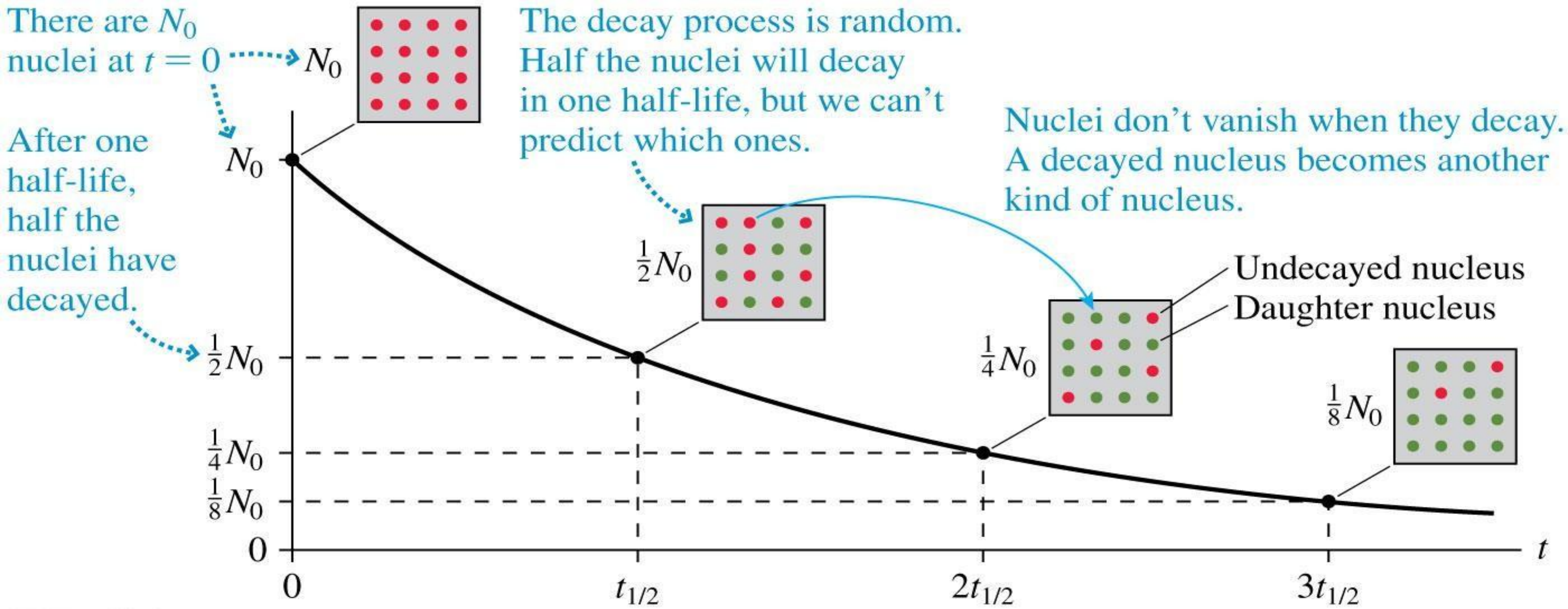
(a)



(b)

Nuclear Decay and Half-Lives

- The figure shows the decay of a sample of radioactive nuclei.



30-8 Half-Life and Rate of Decay

The half-life is the time it takes for half the nuclei in a given sample to decay. It is related to the decay constant:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}. \quad (30\text{-}6)$$

30-8 Half-Life and Rate of Decay

*Mean Life

Sometimes the **mean life** τ of an isotope is quoted, which is defined as $\tau = 1/\lambda$. Then Eq. 30–4 can be written $N = N_0 e^{-t/\tau}$, just as for RC and LR circuits (Chapters 19 and 21 where τ was called the time constant). The mean life of an isotope is then given by (see also Eq. 30–6)

$$\tau = \frac{1}{\lambda} = \frac{T_{\frac{1}{2}}}{0.693}. \qquad \text{[mean life]} \qquad \textbf{(30–7)}$$

The mean life and half-life differ by a factor of 0.693, so confusing them can cause serious error (and has). The radioactive decay law, Eq. 30–5, can then be written as $R = R_0 e^{-t/\tau}$.

Number of decays per second, or decay rate R (activity of the sample):

$$R = \left|\frac{\Delta N}{\Delta t}\right| = R_0 e^{-\lambda t},$$

where $R_0 = |\Delta N/\Delta t|_0$ is the activity at $t = 0$.

Example:

The activity of radioactive source decreases by 5.5% in 31 hours. What is the half-life of this source?

Solution: The original activity is λN_0, so the activity 31.0 hours later is 0.945 λN_0.

$$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow 0.945 = e^{-\lambda t}$$

$$ln0.945 = -\lambda t \Rightarrow \lambda = \frac{-ln0.945}{t}$$

$$T_{1/2} = \frac{ln2}{\lambda} = \frac{ln2}{-ln0.945} t = \frac{ln2}{-ln0.945}(31\ h) = 379.84\ h = 15.8\ days$$

Nuclear Decay and Half-Lives

- If you start with N_0 unstable nuclei, after an interval of time called the *half-life*, you'll have ½ N_0 nuclei remaining.
- The **half-life** $t_{1/2}$ is the average time required for one-half the nuclei to decay.
- The number of nuclei N remaining at time t is

$$N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

Number of atoms remaining after time t → N

Number of atoms at the start, $t = 0$ → N_0

The units for t and $t_{1/2}$ must be the same. → $t/t_{1/2}$

- **No matter how many nuclei there are at any point in time, the number decays by half during the next half-life.**

QuickCheck

100 g of radioactive element X are placed in a sealed box. The half-life of this isotope of X is 2 days. After 4 days have passed, what is the mass of element X in the box?

A. 100 g
B. 50 g
C. 37 g
D. 25 g
E. 0 g

QuickCheck

100 g of radioactive element X are placed in a sealed box. The half-life of this isotope of X is 2 days. After 4 days have passed, what is the mass of element X in the box?

A. 100 g
B. 50 g
C. 37 g
D. 25 g ✔ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ left after 2 half-lives
E. 0 g

Activity

- We can find the variation of activity with time:

$$R = \frac{N}{\tau} = \frac{N_0}{\tau}\left(\frac{1}{2}\right)^{t/t_{1/2}} = \frac{N_0}{\tau}e^{-t/\tau}$$

- N_0/τ is the initial activity R_0 so the decay of activity is

Activity after time t

Activity at the start, $t = 0$

$$R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = R_0 e^{-t/\tau}$$

- The SI unit of activity is the **becquerel**:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay/second or } 1 \text{ s}^{-1}$$

- The **curie** is also used:

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

30-9 Calculations Involving Decay Rates and Half-Life

EXAMPLE 30–9 **Sample activity.** The isotope $^{14}_{6}C$ has a half-life of 5730 yr. If a sample contains 1.00×10^{22} carbon-14 nuclei, what is the activity of the sample?

APPROACH We first use the half-life to find the decay constant (Eq. 30–6), and use that to find the activity, Eq. 30–3b. The number of seconds in a year is $(60)(60)(24)(365\frac{1}{4}) = 3.156 \times 10^7$ s.

SOLUTION The decay constant λ from Eq. 30–6 is

$$\lambda = \frac{0.693}{T_{\frac{1}{2}}} = \frac{0.693}{(5730\text{ yr})(3.156 \times 10^7\text{ s/yr})} = 3.83 \times 10^{-12}\text{ s}^{-1}.$$

From Eqs. 30–3b and 30–5, the activity or rate of decay is

$$R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N = (3.83 \times 10^{-12}\text{ s}^{-1})(1.00 \times 10^{22}) = 3.83 \times 10^{10}\text{ decays/s}.$$

Notice that the graph of Fig. 30–10b starts at this value, corresponding to the original value of $N = 1.0 \times 10^{22}$ nuclei in Fig. 30–10a.

CONCEPTUAL EXAMPLE 30–10 **Safety: Activity versus half-life.** One might think that a short half-life material is safer than a long half-life material because it will not last as long. Is that true?

RESPONSE No. A shorter half-life means the activity is higher and thus more "radioactive" and can cause more biological damage. In contrast, a longer half-life for the same sample size N means a lower activity but we have to worry about it for longer and find safe storage until it reaches a safe (low) level of activity.

EXAMPLE 30–11 **A sample of radioactive $^{13}_{7}N$.** A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min (600 s). (*a*) How many nuclei are present initially? (*b*) What is the rate of decay (activity) initially? (*c*) What is the activity after 1.00 h? (*d*) After approximately how long will the activity drop to less than one per second $\left(= 1\,s^{-1}\right)$?

SOLUTION (*a*) The atomic mass is 13.0, so 13.0 g will contain 6.02×10^{23} nuclei (Avogadro's number). We have only 1.49×10^{-6} g, so the number of nuclei N_0 that we have initially is given by the ratio

$$\frac{N_0}{6.02 \times 10^{23}} = \frac{1.49 \times 10^{-6}\,\text{g}}{13.0\,\text{g}}.$$

Solving for N_0, we find $N_0 = 6.90 \times 10^{16}$ nuclei.

(*b*) From Eq. 30–6,

$$\lambda = 0.693/T_{\frac{1}{2}} = (0.693)/(600\,\text{s}) = 1.155 \times 10^{-3}\,\text{s}^{-1}.$$

Then, at $t = 0$ (see Eqs. 30–3b and 30–5)

$$R_0 = \left|\frac{\Delta N}{\Delta t}\right|_0 = \lambda N_0 = \left(1.155 \times 10^{-3}\,\text{s}^{-1}\right)\left(6.90 \times 10^{16}\right) = 7.97 \times 10^{13}\,\text{decays/s}.$$

(*c*) After 1.00 h = 3600 s, the magnitude of the activity will be (Eq. 30–5)

$$R = R_0 e^{-\lambda t} = \left(7.97 \times 10^{13}\,\text{s}^{-1}\right) e^{-\left(1.155\times10^{-3}\,\text{s}^{-1}\right)(3600\,\text{s})} = 1.25 \times 10^{12}\,\text{s}^{-1}.$$

(*d*) We want to determine the time t when $R = 1.00\,\text{s}^{-1}$. From Eq. 30–5, we have

$$e^{-\lambda t} = \frac{R}{R_0} = \frac{1.00\,\text{s}^{-1}}{7.97 \times 10^{13}\,\text{s}^{-1}} = 1.25 \times 10^{-14}.$$

We take the natural log (ln) of both sides $\left(\ln e^{-\lambda t} = -\lambda t\right)$ and divide by λ to find

$$t = -\frac{\ln\left(1.25 \times 10^{-14}\right)}{\lambda} = 2.77 \times 10^{4}\,\text{s} = 7.70\,\text{h}.$$

Problem 37:

(I) (*a*) What is the decay constant of $^{238}_{92}U$ whose half-life is 4.5×10^9 yr? (*b*) The decay constant of a given nucleus is $3.2 \times 10^{-5}\ s^{-1}$. What is its half-life?

Solution:

(*a*) The decay constant can be found from the half-life, using Eq. 30–6.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.5\times10^{9}\,\text{yr}} = \boxed{1.5\times10^{-10}\,\text{yr}^{-1}} = 4.9\times10^{-18}\,\text{s}^{-1}$$

(*b*) The half-life can be found from the decay constant, using Eq. 30–6.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3.2\times10^{-5}\,\text{s}^{-1}} = 21{,}660\,\text{s} = \boxed{6.0\,\text{h}}$$

Problem 43:

(II) How many nuclei of $^{238}_{92}U$ remain in a rock if the activity registers 420 decays per second?

Solution:

We find the number of nuclei from the activity of the sample, using Eq. 30–3b and Eq. 30–6. The half-life is found in Appendix B.

$$\left|\frac{\Delta N}{\Delta t}\right| = \lambda N = \frac{\ln 2}{T_{1/2}} N \quad \rightarrow$$

$$N = \frac{T_{1/2}}{\ln 2}\left|\frac{\Delta N}{\Delta t}\right| = \frac{(4.468\times10^{9}\ \text{yr})(3.156\times10^{7}\ \text{s/yr})}{\ln 2}(420\ \text{decays/s}) = \boxed{8.5\times10^{19}\ \text{nuclei}}$$

Problem 46:

(II) Calculate the mass of a sample of pure ${}^{40}_{19}\mathrm{K}$ with an initial decay rate of $2.4 \times 10^5\ \mathrm{s}^{-1}$. The half-life of ${}^{40}_{19}\mathrm{K}$ is 1.248×10^9 yr.

Solution:

Find the initial number of nuclei from the initial decay rate (activity) and then the mass from the number of nuclei.

$$\text{initial decay rate} = 2.4 \times 10^5 \text{ decays/s} = \lambda N_0 = \frac{\ln 2}{T_{1/2}} N_0 \quad \rightarrow$$

$$N_0 = \frac{T_{1/2}}{\ln 2}(2.4\times10^5\ \mathrm{s}^{-1}) = \frac{(1.248\times10^9\ \mathrm{yr})(3.156\times10^7\ \mathrm{s/yr})}{\ln 2}(2.4\times10^5\ \mathrm{s}^{-1}) = 1.364\times10^{22}\ \text{nuclei}$$

$$m = N_0 \frac{(\text{atomic weight})\ \mathrm{g/mol}}{6.02\times10^{23}\ \text{nuclei/mol}} = (1.364\times10^{22}\ \text{nuclei})\frac{(39.963998\ \mathrm{g})}{(6.02\times10^{23})} = \boxed{0.91\ \mathrm{g}}$$

Problem 49:

(II) The activity of a sample drops by a factor of 6.0 in 9.4 minutes. What is its half-life?

Solution:

$$R = \tfrac{1}{6} R_0 = R_0 e^{-\lambda t} \rightarrow \tfrac{1}{6} = e^{-\lambda t} \rightarrow \ln\left(\tfrac{1}{6}\right) = -\lambda t = -\frac{\ln 2}{T_{1/2}} t \rightarrow$$

$$T_{1/2} = -\frac{\ln 2}{\ln\left(\tfrac{1}{6}\right)} t = -\frac{\ln 2}{\ln\left(\tfrac{1}{6}\right)} (9.4\ \text{min}) = \boxed{3.6\ \text{min}}$$