Chapter 3:

Kinematics in Two Dimensions: Vectors

- *Department of Physics*
- *The University of Jordan*

Vectors

- General discussion.

  Vector ≡ A quantity with **magnitude & direction.**

  Scalar ≡ A quantity with **magnitude only.**

- Here: We mainly deal with

  Displacement ≡ D & **Velocity ≡ v**

  Our discussion is valid for any vector!

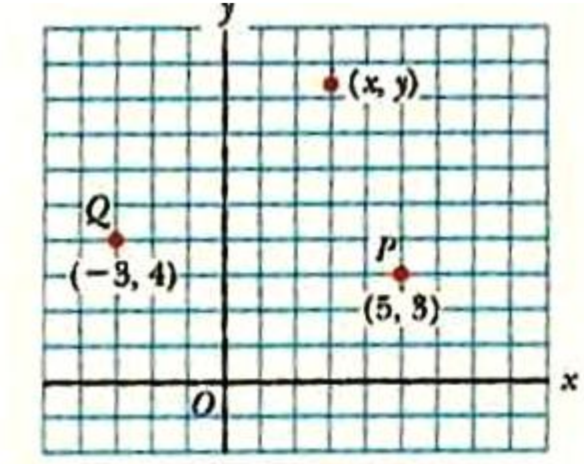
- The vector part of the chapter has a lot of math! It requires detailed knowledge of trigonometry.

- **Problem Solving**

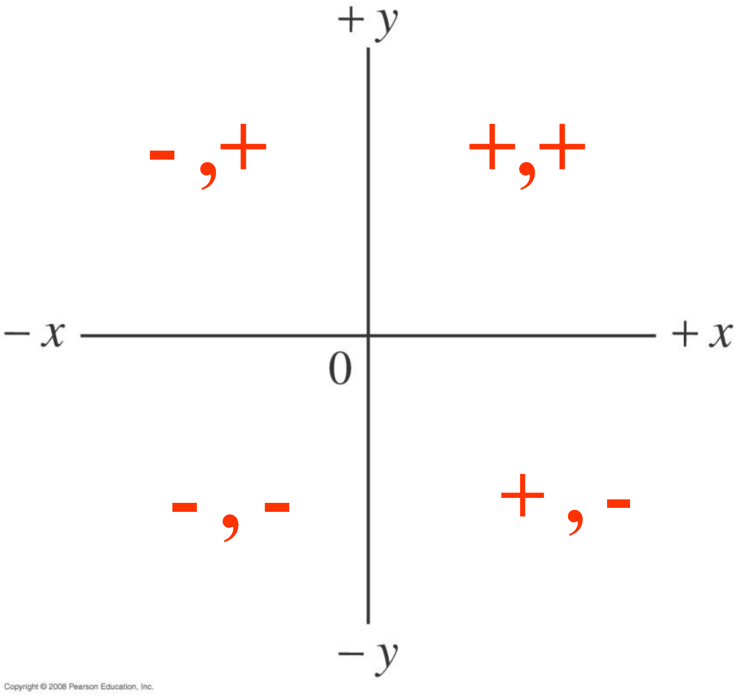
  A ***diagram or sketch*** is helpful & ***vital***! I don't see how it is possible to solve a vector problem without a diagram!

Coordinate Systems

- **Rectangular or Cartesian Coordinates**
  - "Standard" coordinate axes.
  - Point in the plane is **(x,y)**
  - Note, if its convenient could reverse + & -



Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).



Standard set of xy coordinate axes

Vector & Scalar Quantities

Vector

≡ A quantity with **magnitude & direction.**

Scalar

≡ A quantity with **magnitude only.**

- **Equality of two vectors**

2 vectors, **A** & **B** **,** **A = B** means **A** & **B** have the same **magnitude & direction.**

Sect. 3-2: Vector Addition, Graphical Method

- **Addition of scalars:** "Normal" arithmetic!
- **Addition of vectors:** Not so simple!
- **Vectors in the same direction:**
  - Can also use simple arithmetic

Example: Travel 8 km East on day 1, 6 km East on day 2.

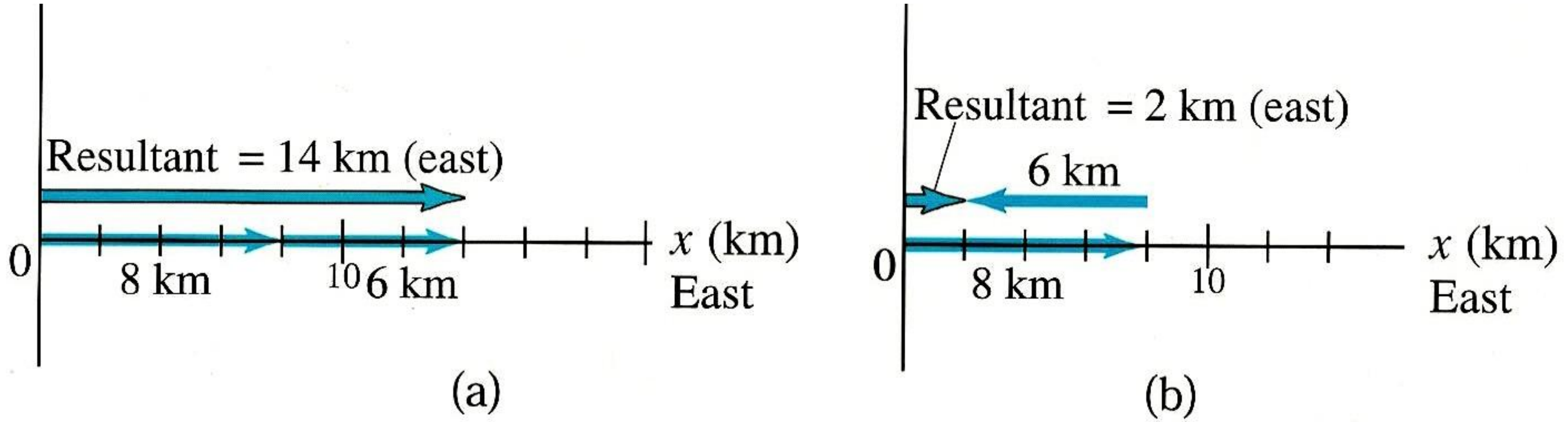
Displacement = 8 km + 6 km = 14 km East

Example: Travel 8 km East on day 1, 6 km West on day 2.

Displacement = 8 km - 6 km = 2 km East

"Resultant" = Displacement

- **Adding vectors in the same direction:**



Graphical Method

- For 2 vectors **NOT** along same line, adding is more complicated:

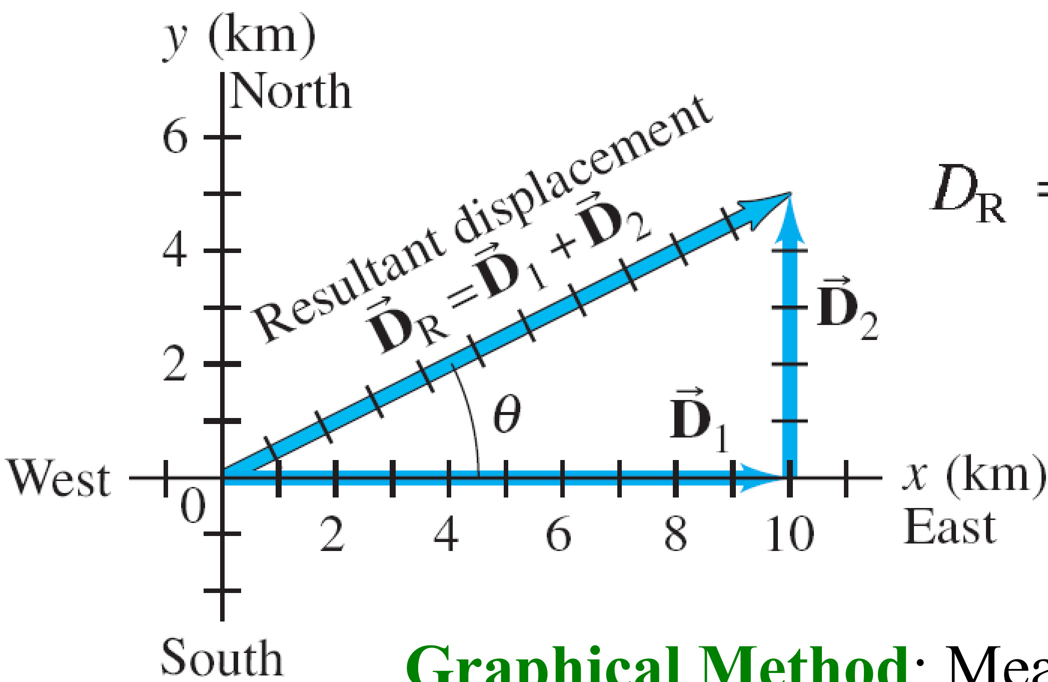
  Example: $\mathbf{D_1}$ = 10 km East, $\mathbf{D_2}$ = 5 km North. What is the **resultant** (final) displacement?

- **2 methods of vector addition:**
  - Graphical (2 methods of this also!)
  - Analytical (***TRIGONOMETRY***)

- 2 vectors **NOT** along same line: $\mathbf{D_1}$ = 10 km E, $\mathbf{D_2}$ = 5 km N.

Resultant = $\mathbf{D_R = D_1 + D_2}$ = ?

In this special case ***ONLY***, $\mathbf{D_1}$ is perpendicular to $\mathbf{D_2}$.
So, we can use the ***Pythagorean Theorem*.**



$$D_R = \sqrt{D_1^2 + D_2^2} = \mathbf{11.2\ km}$$

Note! $\mathbf{D_R < D_1 + D_2}$
(scalar addition)

Graphical Method: Measure.
Find **$\mathbf{D_R}$ = 11.2 km, θ = 27º N of E**

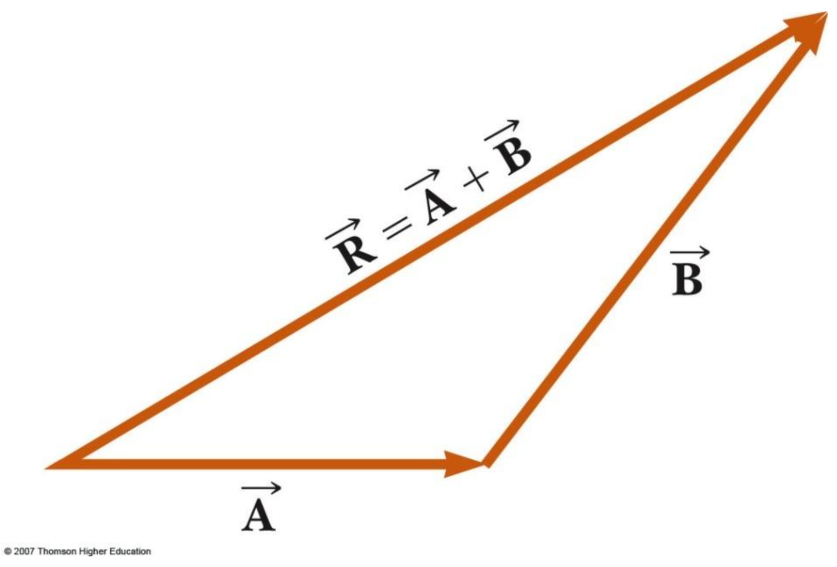
- Example illustrates general rules ("tail-to-tip" method of graphical addition). Consider **R** = **A** + **B**

  1. Draw **A** & **B** to scale. (1 Newton ➔ 1 cm)

  2. Place tail of **B** at tip of **A**

  3. Draw arrow from tail of **A** to tip of **B**

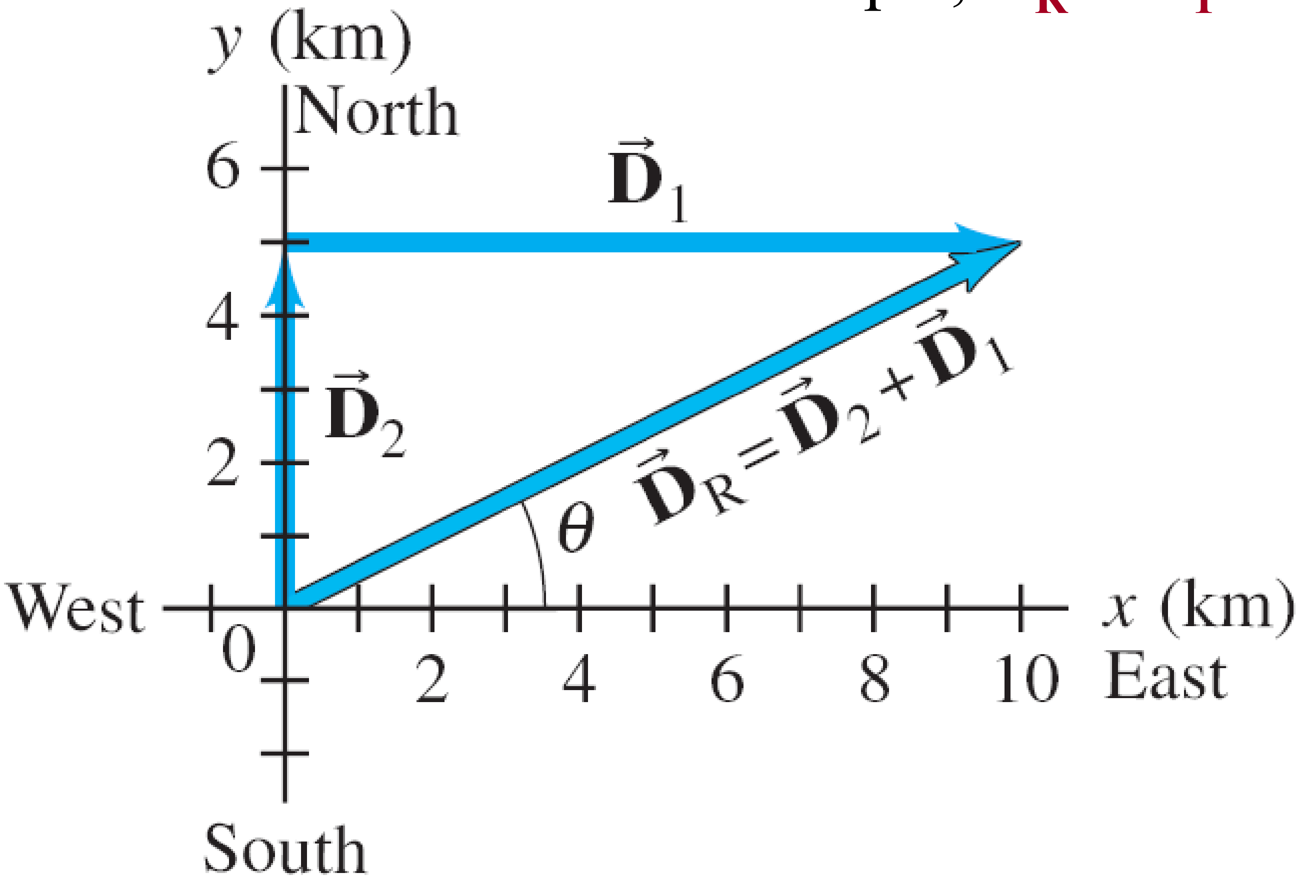
This arrow is the resultant R (measure length & the angle it makes with the **x**-axis)



Order isn't important! Adding the vectors in the opposite order gives the same result:

$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1).$$

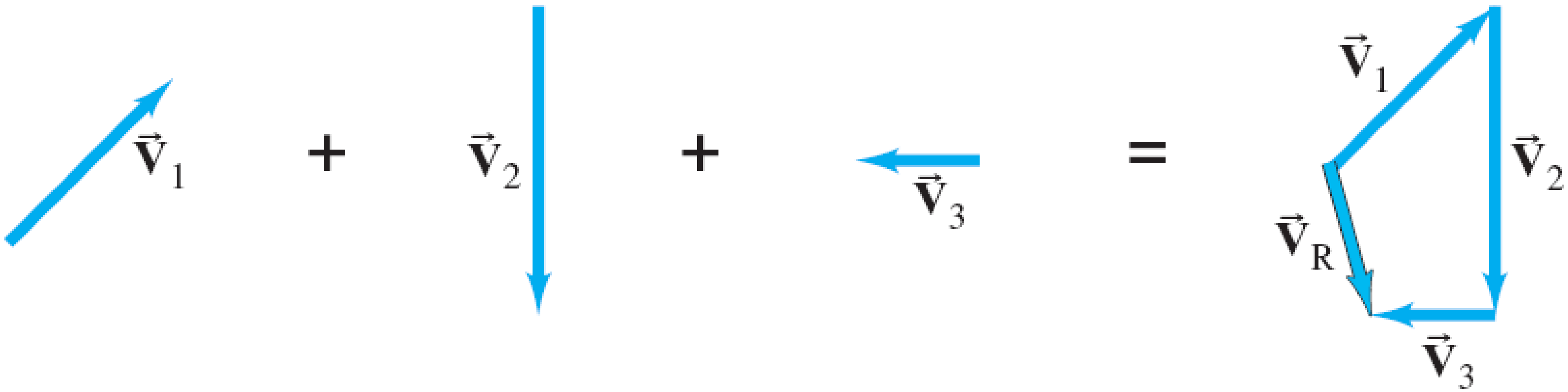
In the example, $\mathbf{D_R = D_1 + D_2 = D_2 + D_1}$



Graphical Method

- Adding (3 or more) vectors

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$$



Even if the vectors are not at right angles, they can be added graphically by using the tail-to-tip method.

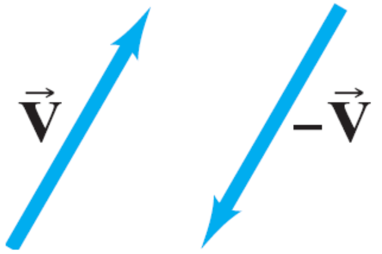
Subtraction of Vectors

- First, **define the negative of a vector:**

  - V ≡ vector with the same magnitude (size) as **V** but with opposite direction.

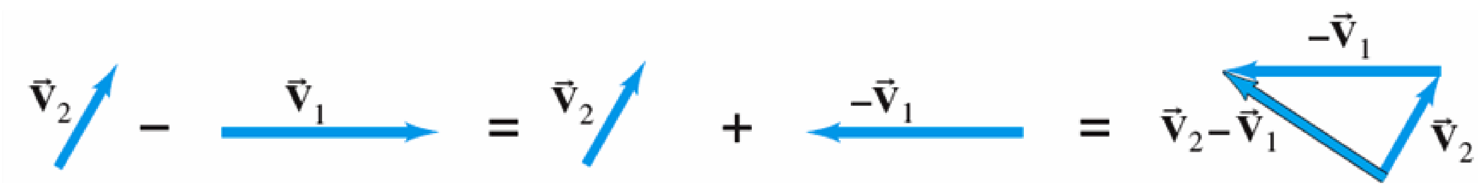
  Math: $\mathbf{V} + (-\mathbf{V}) \equiv \mathbf{0}$

  Then add the negative vector.



- For 2 vectors, $\mathbf{V}_1$ & $\mathbf{V}_2$:

$$\mathbf{V}_1 - \mathbf{V}_2 \equiv \mathbf{V}_1 + (-\mathbf{V}_2)$$



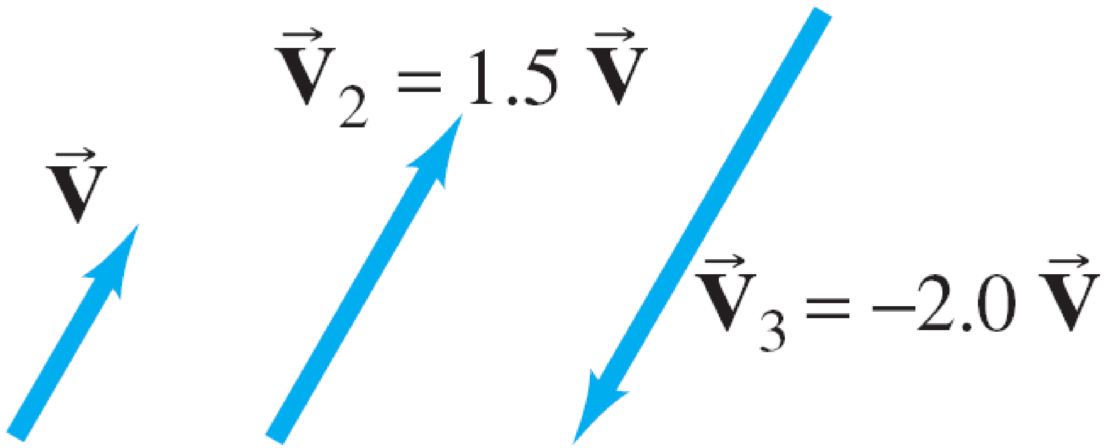
Multiplication by a Scalar

A vector **V** can be multiplied by a scalar **c**

$$\mathbf{V'} = c\mathbf{V}$$

V' ≡ vector with magnitude **cV** & same direction as **V**

If **c** is negative, the resultant is in the opposite direction.



Example

- A two part car trip. First, displacement **A = 20 km due North**. Then, displacement **B = 35 km 60º West of North**. Figure. Find (graphically) resultant displacement vector **R** (magnitude & direction).

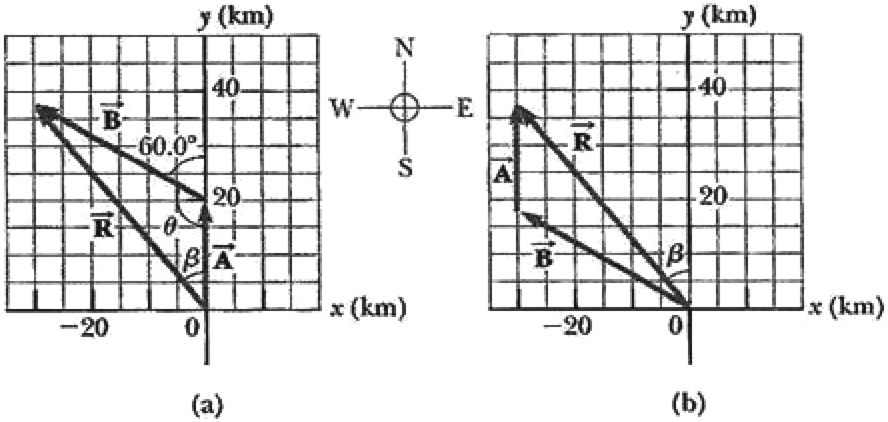
R = A + B

Use ***ruler & protractor***

to find **length of R**, **angle β.**

Length = 48.2 km

β = 38.9º



(a) Graphical method for finding the resultant displacement vector $\vec{R} = \vec{A} + \vec{B}$. (b) Adding the vectors in reverse order ($\vec{B} + \vec{A}$) gives the same result for $\vec{R}$.

The Analytic Method of Addition

Resolution of vectors into components:

***YOU MUST KNOW* &**

UNDERSTAND

***TRIGONOMETERY* TO**

UNDERSTAND THIS!!!!

Vector Components

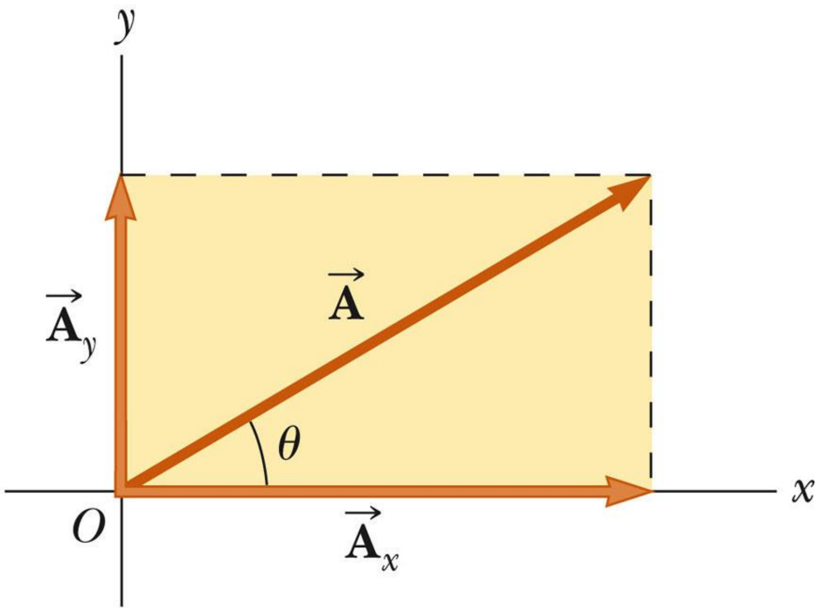
- Any vector can be expressed as the sum of two other vectors, called its **components**. Usually, the other vectors are chosen so that they are perpendicular to each other.
- Consider the vector **V** in a plane (say, the **xy** plane)
- We can express **V** in terms of ***COMPONENTS*** $\mathbf{V_x}$, $\mathbf{V_y}$
- Finding ***THE COMPONENTS*** $\mathbf{V_x}$ **&** $\mathbf{V_y}$ is ***EQUIVALENT*** to finding 2 mutually perpendicular vectors which, when added (with vector addition) will give **V**.
- That is, find $\mathbf{V_x}$ **&** $\mathbf{V_y}$ such that

$$\mathbf{V} \equiv \mathbf{V_x} + \mathbf{V_y} \qquad (\mathbf{V_x} \parallel \text{x axis}, \mathbf{V_y} \parallel \text{y axis})$$

Finding Components ≡

"Resolving into Components"

- Mathematically, a ***component is a projection*** of a vector along an axis
  - Any vector can be completely described by its components

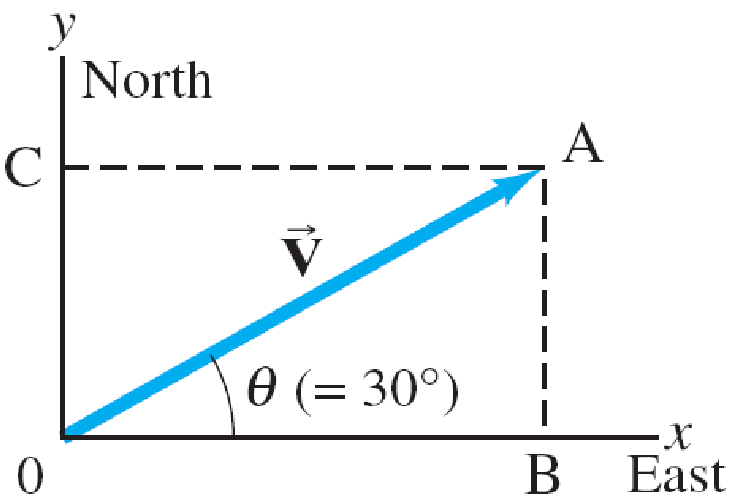


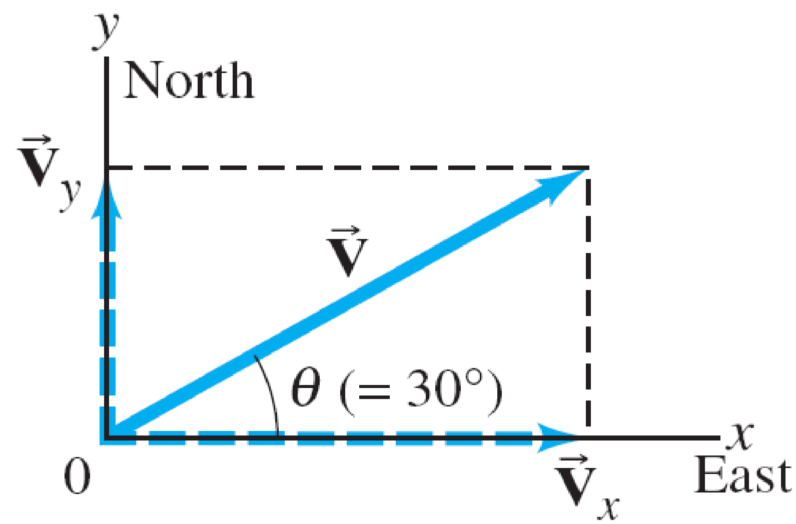
(a)

- It is useful to use ***rectangular components***
  - These are the projections of the vector along the x- and y-axes

V is **resolved into components: V_x & V_y**

$V \equiv V_x + V_y$ **(V_x || x axis, V_y || y axis)**





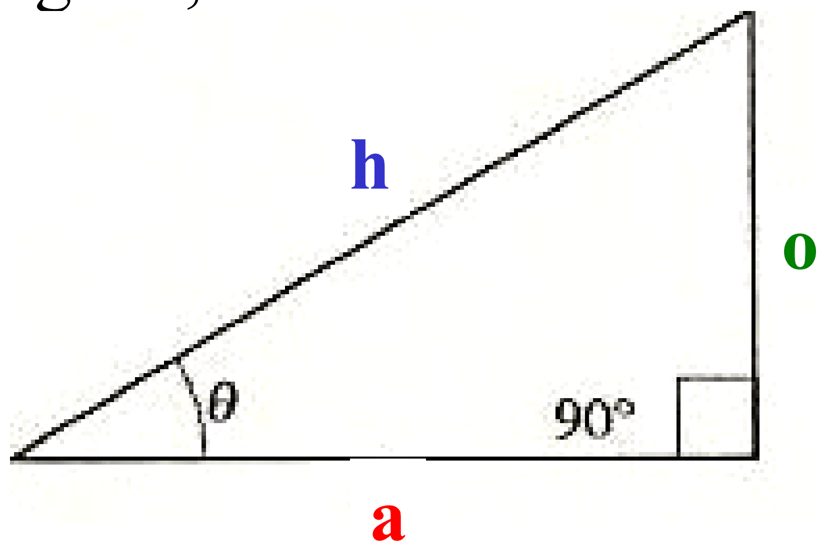
By ***the parallelogram method***, clearly

THE VECTOR SUM IS: $V = V_1 + V_2$

In 3 dimensions, we also need a V_z.

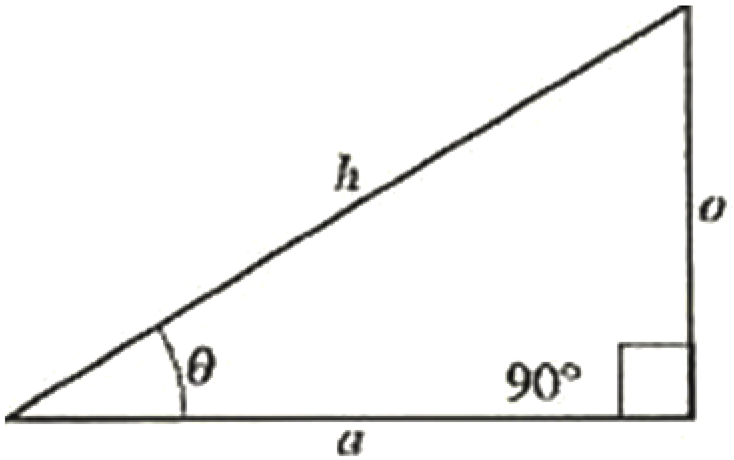
A Brief Trig Review

- Adding vectors in 2 & 3 dimensions using components requires ***TRIG FUNCTIONS***
- ***HOPEFULLY, A REVIEW!!***
  - See also **Appendix A**!!
- Given any angle **θ**, we can construct a right triangle:



Hypotenuse ≡ h, Adjacent side ≡ a, Opposite side ≡ o

- Define the ***trig functions*** in terms of **h, a, o:**



$$\sin\theta = \frac{o}{h}$$ **= (opposite side)/(hypotenuse)**

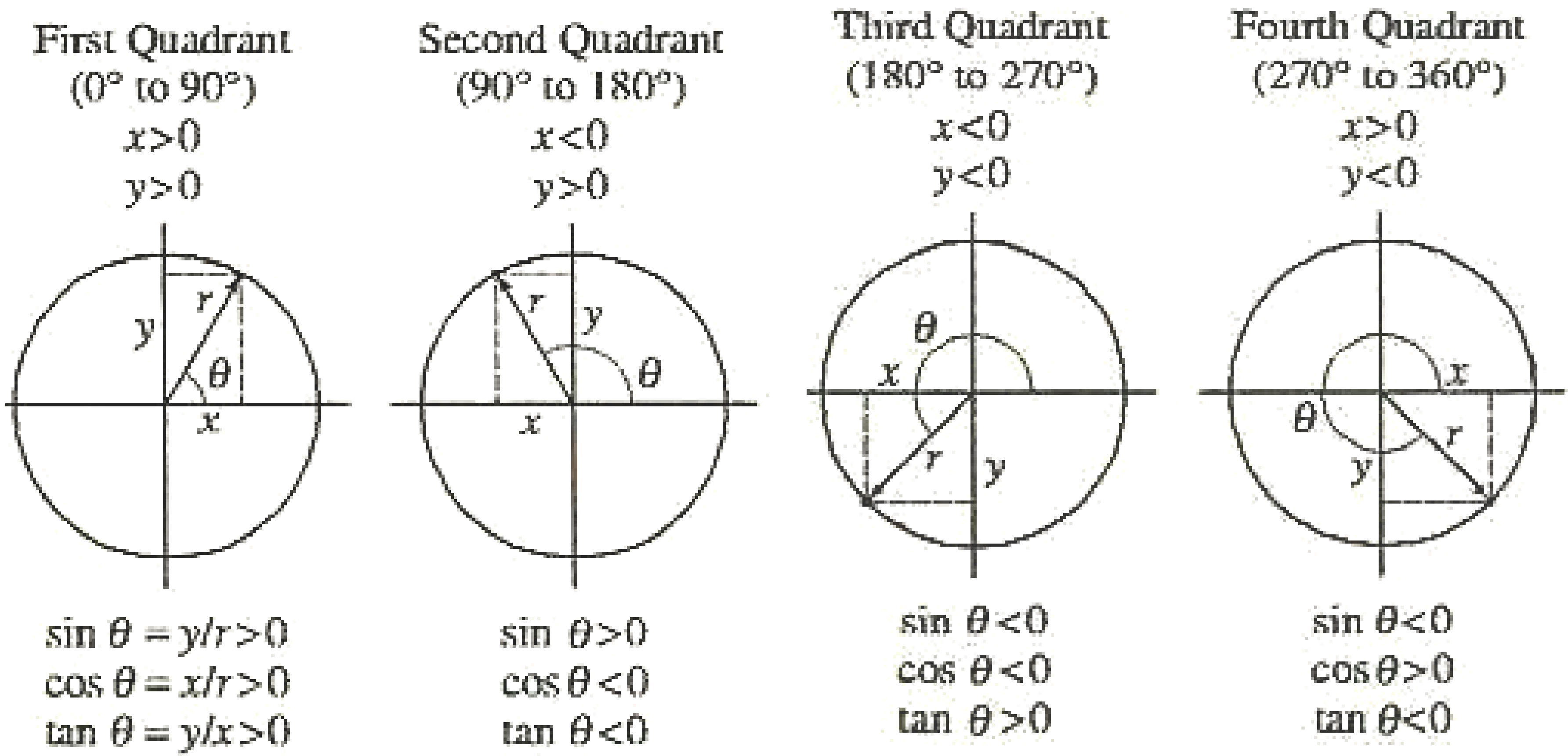
$$\cos\theta = \frac{a}{h}$$ **= (adjacent side)/(hypotenuse)**

$$\tan\theta = \frac{o}{a} = \frac{\sin\theta}{\cos\theta}$$ **= (opposite side)/(adjacent side)**

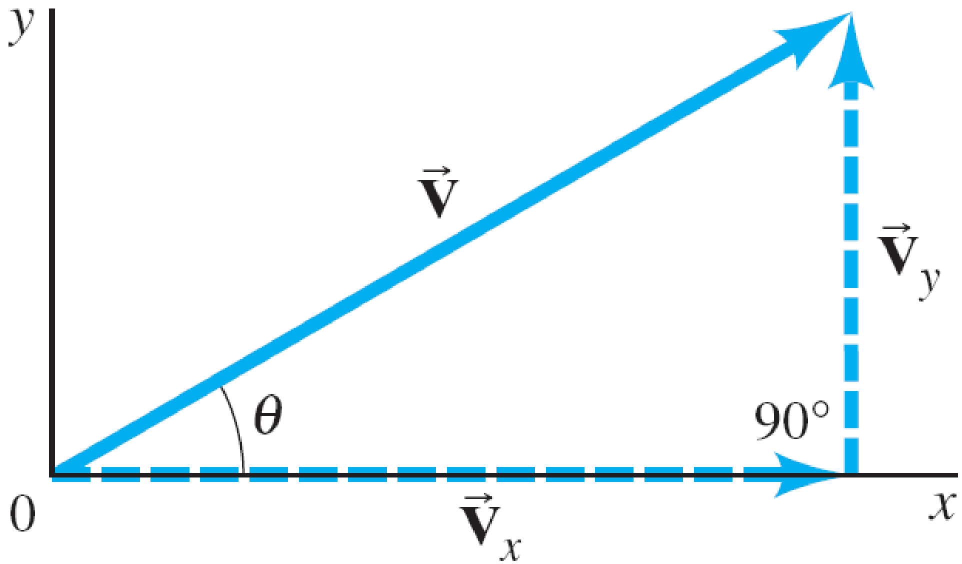
$$a^2 + o^2 = h^2$$ **[Pythagorean theorem]**

Signs of the sine, cosine & tangent

- **Trig identity: $\tan(\theta) = \sin(\theta)/\cos(\theta)$**



Trig Functions to Find Vector Components



We can use all of this to ***add vectors analytically!***

$$\sin\theta = \frac{V_y}{V} \qquad V_y = V\sin\theta$$

$$\cos\theta = \frac{V_x}{V} \qquad V_x = V\cos\theta$$

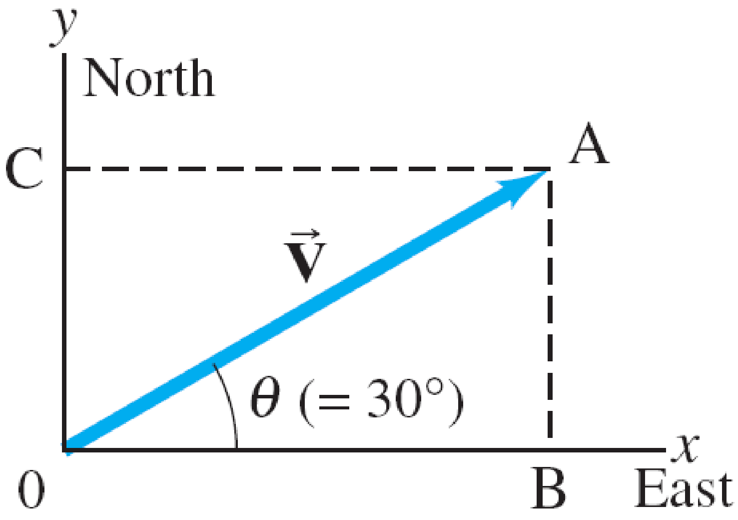
$$\tan\theta = \frac{V_y}{V_x} \qquad V = \sqrt{V_x^2 + V_y^2}$$

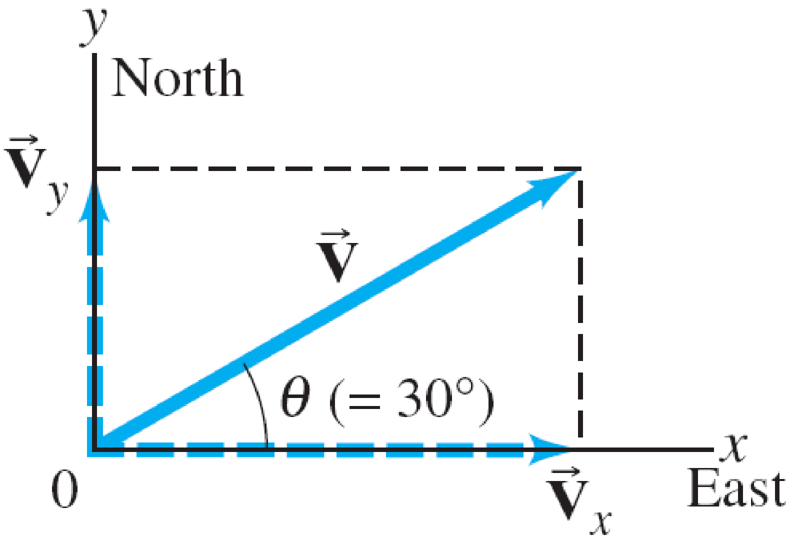
$$V^2 = V_x^2 + V_y^2$$

[Pythagorean Theorem]

Example

V = Displacement = 500 m, 30º N of E





$$V_y = V \sin\theta = 250\,\text{m}$$

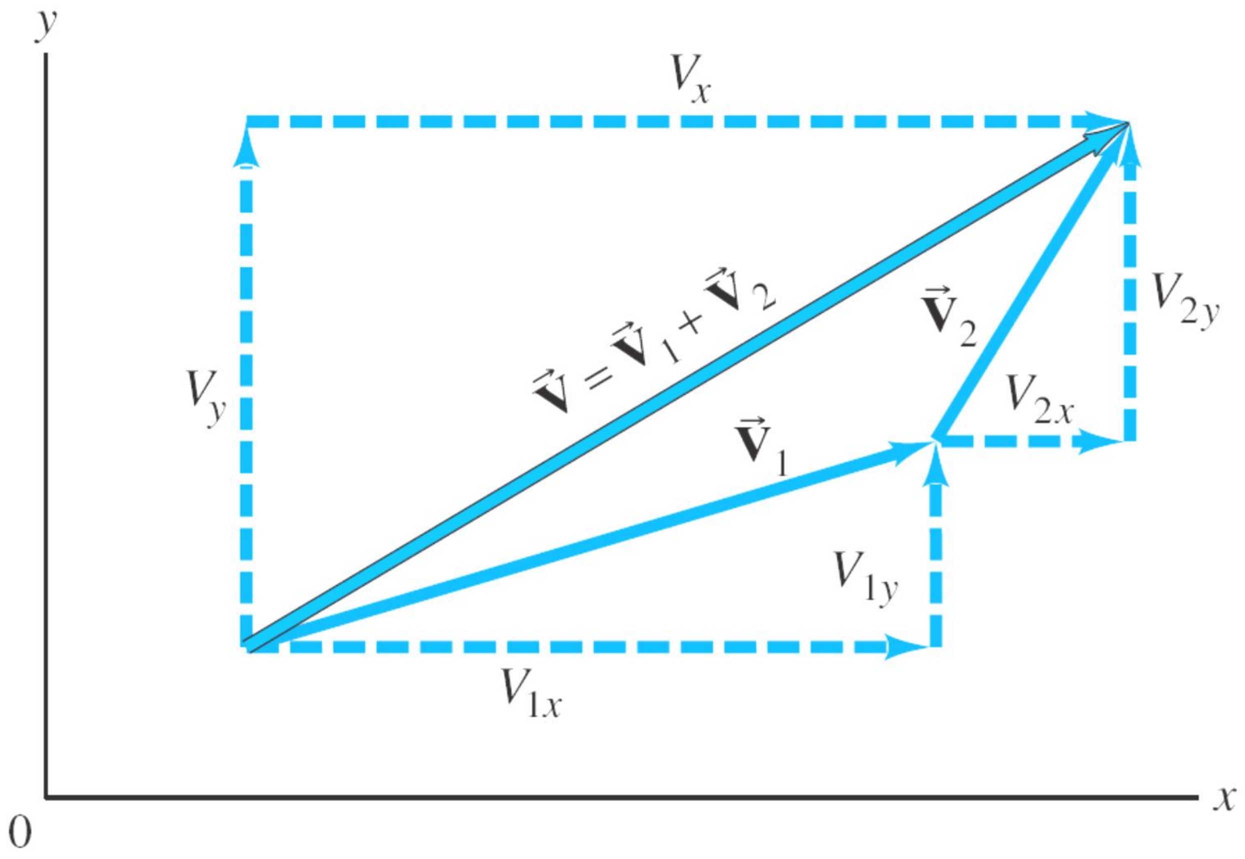
$$V_x = V \cos\theta = 433\,\text{m}$$

$$V = \sqrt{V_x^2 + V_y^2} = 500\,\text{m}$$

Example

Consider 2 vectors, **V_1 & V_2.** We want **$V = V_1 + V_2$**

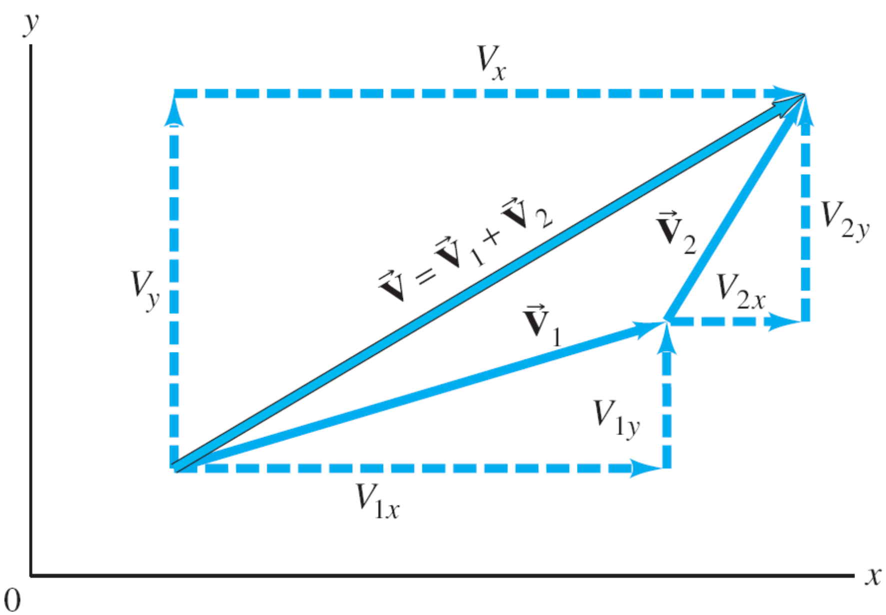
- ***Note:*** The components of each vector are really one-dimensional vectors, so they can be added arithmetically.



We want the vector sum $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$

"Recipe" for adding 2 vectors using trig & components:

1. ***Sketch*** a diagram to ***roughly*** add the vectors graphically. Choose **x** & **y** axes.
2. Resolve each vector into **x** & **y** components using sines & cosines. That is, find $\mathbf{V}_{1x}$, $\mathbf{V}_{1y}$, $\mathbf{V}_{2x}$, $\mathbf{V}_{2y}$. ($\mathbf{V}_{1x} = \mathbf{V}_1 \cos \theta_1$, etc.)
4. Add the components in each direction. ($\mathbf{V}_x = \mathbf{V}_{1x} + \mathbf{V}_{2x}$, etc.)
5. Find the length & direction of **V**, use:

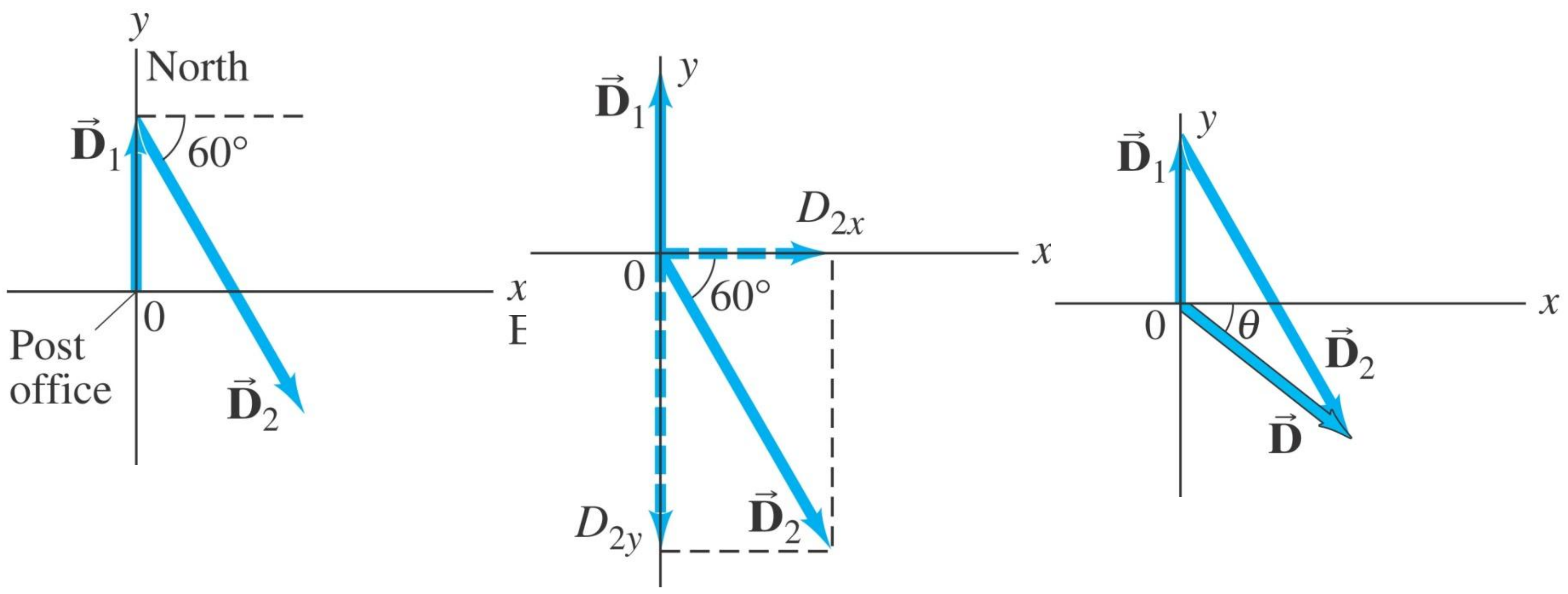


$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan \theta = \frac{V_y}{V_x}$$

Example 3-2

A rural mail carrier leaves the post office & drives **22.0 km** in a northerly direction. She then drives in a direction **60.0°** south of east for **47.0 km**. What is her displacement from the post office?



Example 3-2

$D_{1x} = 0$, $D_{1y} = 22.0$ km

$D_{2x} = (47.0 \text{ km})(\cos 60°) = (47.0 \text{ km})(0.500) = 23.5 \text{ km}$

$D_{2y} = -(47.0 \text{ km})(\sin 60°) = -(47.0 \text{ km})(0.866) = -40.7 \text{ km}$

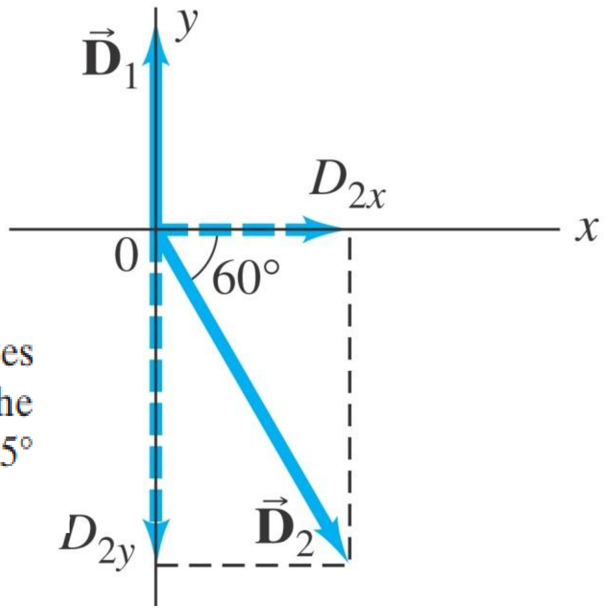
$D_{Rx} = D_{1x} + D_{2x} = 0 \text{ km} + 23.5 \text{ km} = 23.5 \text{ km}$

$D_{Ry} = D_{1y} + D_{2y} = 22.0 \text{ km} + (-40.7 \text{ km}) = -18.7 \text{ km}.$

$$D_R = \sqrt{D_{Rx}^2 + D_{Ry}^2} = \sqrt{(23.5 \text{ km})^2 + (-18.7 \text{ km})^2} = 30.0 \text{ km}$$

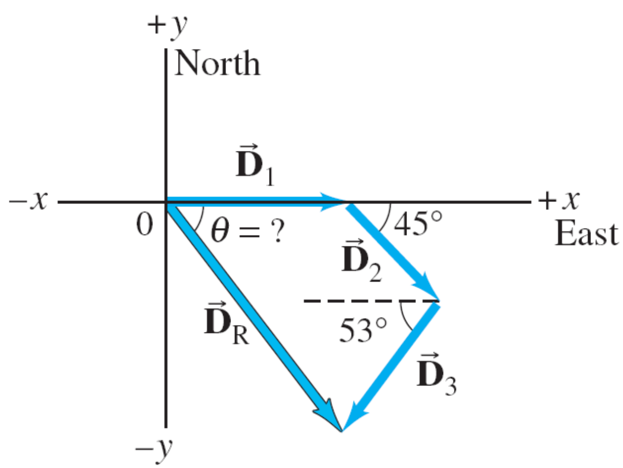
$$\tan\theta = \frac{D_{Ry}}{D_{Rx}} = \frac{-18.7 \text{ km}}{23.5 \text{ km}} = -0.796.$$

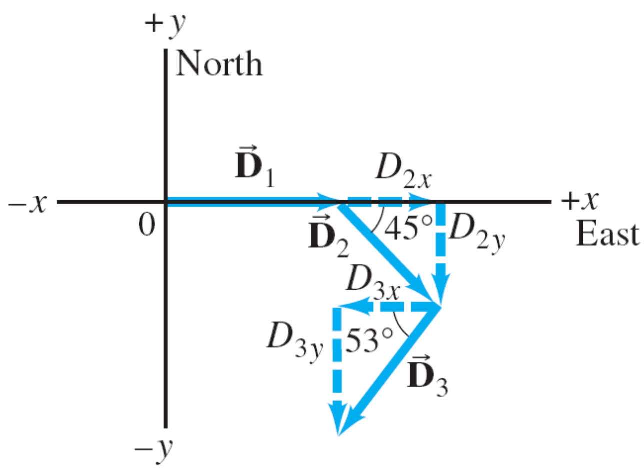
A calculator with a key labeled INV TAN, or ARC TAN, or TAN^{-1} gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 3–15c. So, the resultant displacement is 30.0 km directed at 38.5° in a southeasterly direction.



Example 3-3

A plane trip involves 3 legs, with 2 stopovers: **1)** Due east for **620 km**, **2)** Southeast (**45°**) for **440 km**, **3)** **53°** south of west, for **550 km**. Calculate the plane's total displacement.





Vector	**Components** *x* **(km)**	*y* **(km)**
$\vec{D}_1$	620	0
$\vec{D}_2$	311	−311
$\vec{D}_3$	−331	−439
$\vec{D}_R$	600	−750

Problem Solving

Here is a brief summary of how to add two or more vectors using components:

1. Draw a diagram, adding the vectors graphically.
2. Choose x and y axes. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors so that vector will have only one component.)
3. Resolve each vector into its x and y components, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. Calculate each component (when not given) using sines and cosines. If θ_1 is the angle vector $\mathbf{V}_1$ makes with the x axis, then:

$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$

   Pay careful attention to signs: any component that points along the negative x or y axis gets a negative sign.

5. Add the x components together to get the x component of the resultant. Ditto for y:

$$V_x = V_{1x} + V_{2x} + \text{any others}$$

$$V_y = V_{1y} + V_{2y} + \text{any others.}$$

   This is the answer: the components of the resultant vector.

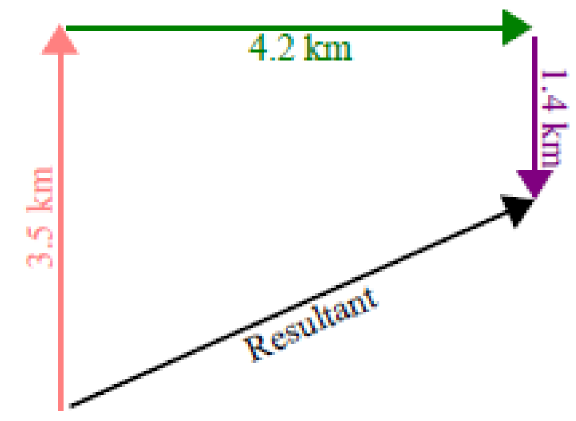
6. If you want to know the magnitude and direction of the resultant vector, use Eqs. 3–4:

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}.$$

   The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

You cannot solve a vector problem without drawing a diagram!

Example : Man goes for a walk to see if he can find a grocery store to buy a Cigarette . He first walks 3.5 km [N], then 4.2 km [E] , and finally 1.4 km [S] before getting to the 7the store. Determine the displacement of the person.



Solution

$$c^2 = a^2 + b^2$$
$$c^2 = 2.1^2 + 4.2^2$$
$$c = 4.695743$$
$$c = 4.7\,km$$

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{4.2}{2.1}$$
$$\theta = 63.43495^o$$
$$\theta = 63^o$$

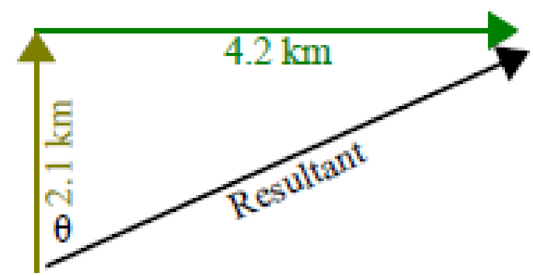


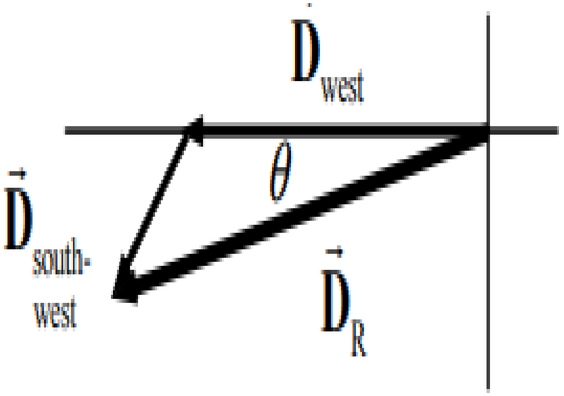
Illustration 3: Adding two vectors to get a resultant.

The person walked a displacement of 4.7 km [N63°E].

Problem 1:

A car is driven 225 km west and then 98 km southwest (45°). What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.

Solution

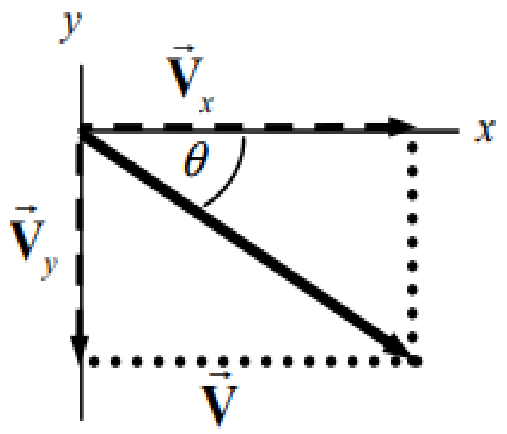


The resultant vector displacement of the car is given by $\vec{D}_R = \vec{D}_{west} + \vec{D}_{south\text{-}west}$. The westward displacement is $225\ \text{km} + (98\ \text{km})\cos 45° = 294.3\ \text{km}$ and the southward displacement is $(98\ \text{km})\sin 45° = 69.3\ \text{km}$. The resultant displacement has a magnitude of $\sqrt{294.3^2 + 69.3^2} = \boxed{302\ \text{km}}$. The direction is $\theta = \tan^{-1} 69.3/294.3 = \boxed{13°\ \text{south of west}}$.

Problem 3:

If Vx = 9.80 units and Vy = -6.40 units determine the magnitude and direction of $\vec{V}$

Solution



Given that $V_x = 9.80$ units and $V_y = -6.40$ units, the magnitude of $\vec{V}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{9.80^2 + (-6.40)^2} = \boxed{11.70 \text{ units}}$. The direction is given by $\theta = \tan^{-1}\frac{-6.40}{9.80} = \boxed{-33.1°}$, or 33.1° below the positive x axis.

Problem 8:

Very important

8. (II) An airplane is traveling 835 km/h in a direction 41.5° west of north (Fig. 3–34). (*a*) Find the components of the velocity vector in the northerly and westerly directions. (*b*) How far north and how far west has the plane traveled after 1.75 h?

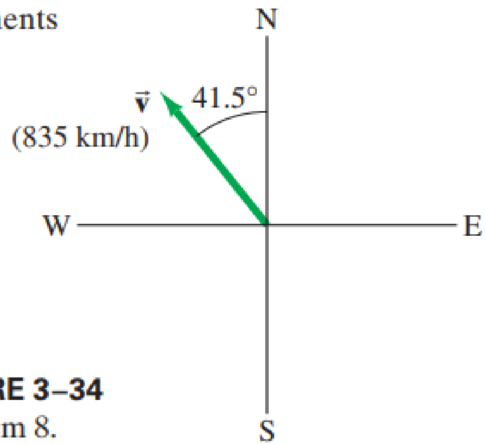


FIGURE 3–34
Problem 8.

Solution

(*a*) $v_{\text{north}} = (835\text{ km/h})(\cos 41.5°) = \boxed{625\text{ km/h}}$ $\quad v_{\text{west}} = (835\text{ km/h})(\sin 41.5°) = \boxed{553\text{ km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (625\text{ km/h})(1.75\text{ h}) = \boxed{1090\text{ km}}$

$\Delta d_{\text{west}} = v_{\text{west}} t = (553\text{ km/h})(1.75\text{ h}) = \boxed{968\text{ km}}$

Problem 12:

12. (II) For the vectors shown in Fig. 3–35, determine (*a*) $\vec{\mathbf{B}} - 3\vec{\mathbf{A}}$, (*b*) $2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}$.

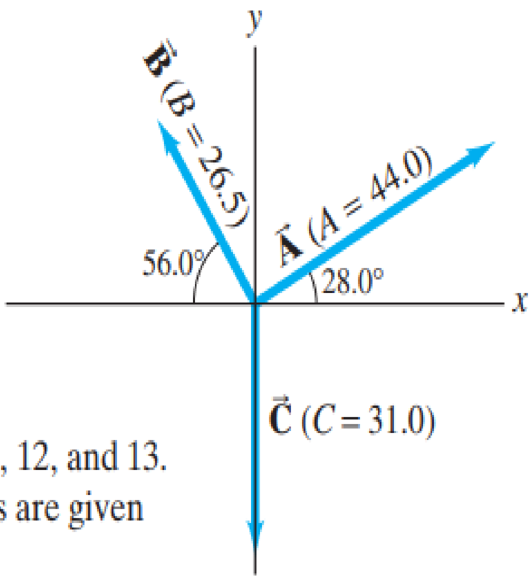


FIGURE 3–35
Problems 9, 10, 11, 12, and 13. Vector magnitudes are given in arbitrary units.

Solution

$A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270^0 = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(*a*) $(\vec{\mathbf{B}} - 3\vec{\mathbf{A}})_x = -14.82 - 3(38.85) = -131.37 \quad (\vec{\mathbf{B}} - 3\vec{\mathbf{A}})_y = 21.97 - 3(20.66) = -40.01$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\left|\vec{\mathbf{B}} - 3\vec{\mathbf{A}}\right| = \sqrt{(-131.37)^2 + (-40.01)^2} = 137.33 \approx \boxed{137}$$

$$\theta = \tan^{-1}\frac{-40.01}{-131.37} = \boxed{16.9° \text{ below } -x \text{ axis}}$$

(*b*) $(2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}})_x = 2(38.85) - 3(-14.82) + 2(0.0) = 122.16$

$(2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}})_y = 2(20.66) - 3(21.97) + 2(-31.0) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2 + (-86.59)^2} = \boxed{149.7} \quad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3° \text{ below } +x \text{ axis}}$$