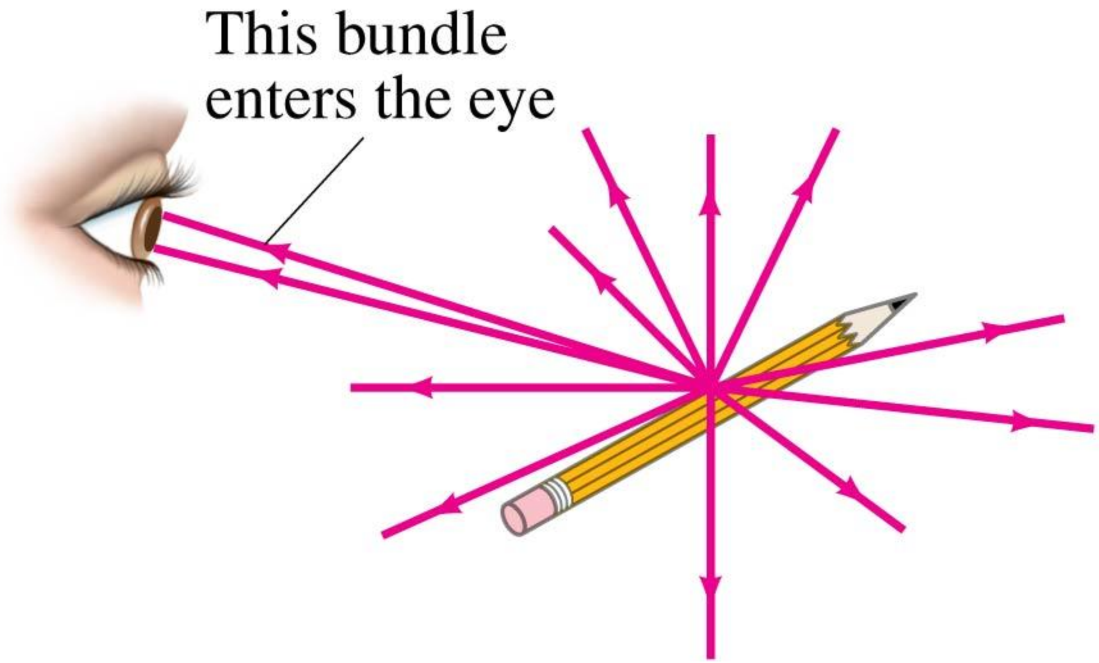
Chapter 23:

Light: Geometric Optics

- *Department of Physics*
- *The University of Jordan*

23-1 The Ray Model of Light

Light very often travels in straight lines. We represent light using rays, which are straight lines emanating from an object. This is an idealization, but is very useful for geometric optics.



23-4 Index of Refraction

In general, light slows somewhat when traveling through a medium. The index of refraction of the medium is the ratio of the speed of light in vacuum to the speed of light in the medium:

$$n = \frac{c}{v}.$$

For example, since n=1.33 for water, the speed of light in water is

$$v = \frac{c}{n} = \frac{(3.00 \times 10^8\,\mathrm{m/s})}{1.33} = 2.26 \times 10^8\,\mathrm{m/s}.$$

TABLE 23–1 Indices of Refraction†

Material	$n = \frac{c}{v}$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flint	1.58
Plastic	
Acrylic, Lucite, CR-39	1.50
Polycarbonate	1.59
"High-index"	1.6–1.7
Sodium chloride	1.53
Diamond	2.42

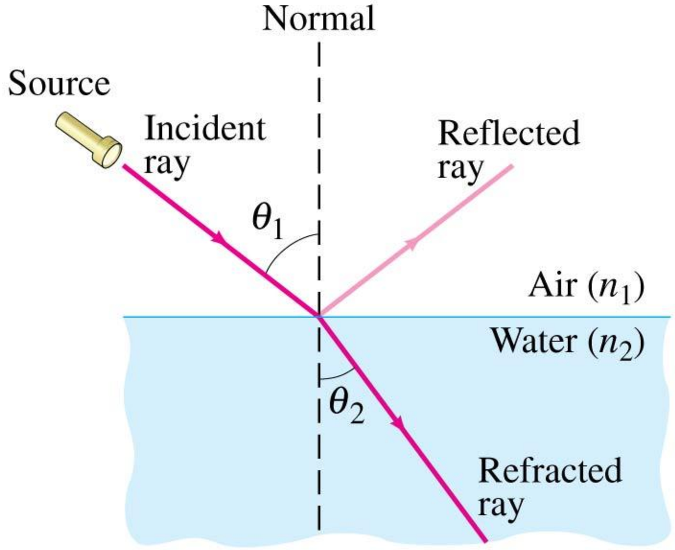
† $\lambda = 589$ nm.

Problem 26:

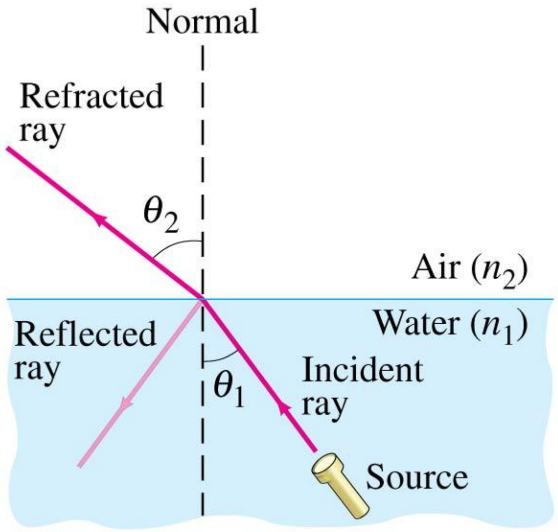
The speed of light in a certain substance is 82% of its value in water. What is the index of refraction of that substance?

23-5 Refraction: Snell's Law

Light changes direction when crossing a boundary from one medium to another. This is called refraction, and the angle the outgoing ray makes with the normal is called the angle of refraction.



(a) $n_2 > n_1$: Ray bends toward ⊥



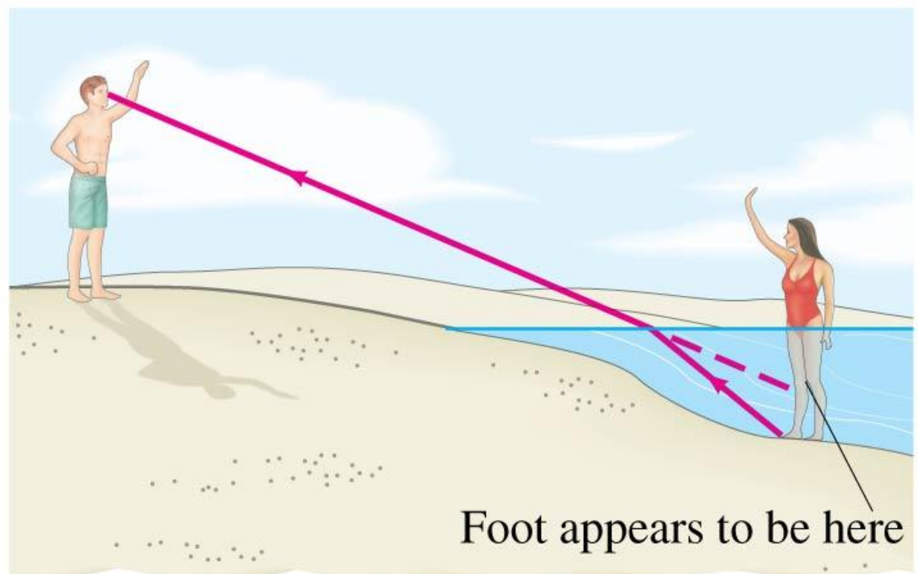
(b) $n_1 > n_2$: Ray bends away from ⊥

23-5 Refraction: Snell's Law

Refraction is what makes objects half-submerged in water look odd.

(a)

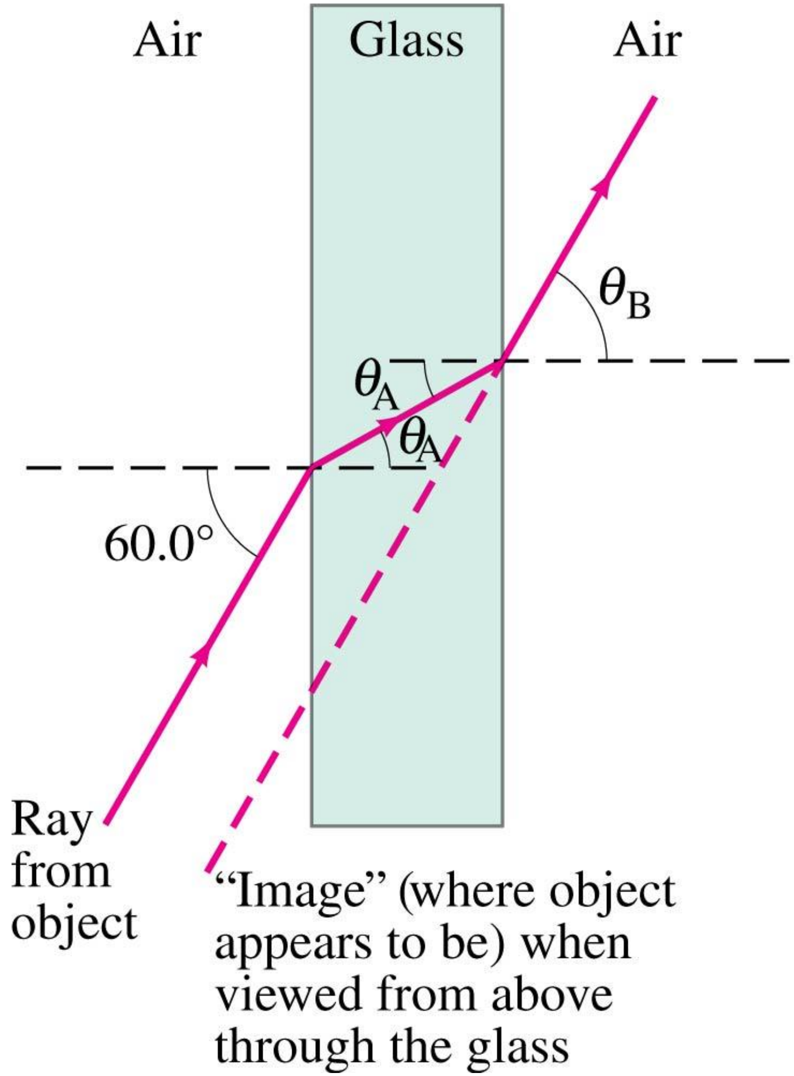
(b)



23-5 Refraction: Snell's Law

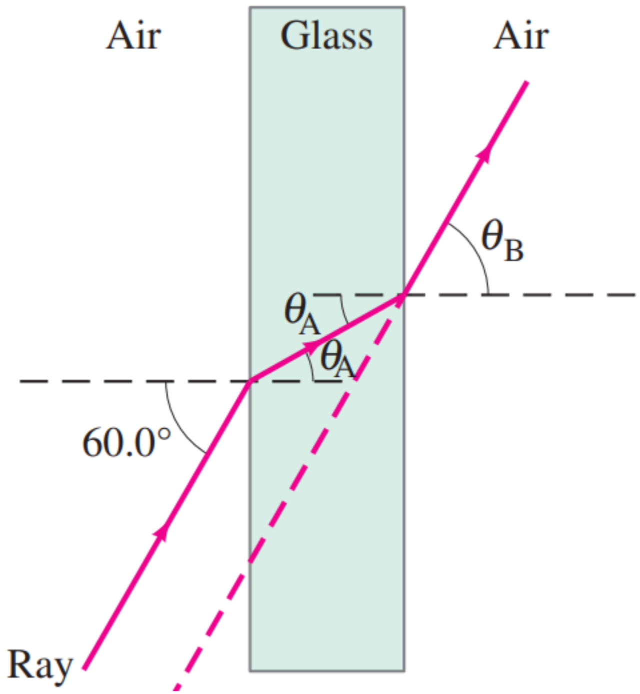
The angle of refraction depends on the indices of refraction, and is given by Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 . \qquad (23\text{-}5)$$



Example 23-8:

Light traveling in air strikes a flat piece of uniformly thick glass at an incident angle of 60.0°, as shown in Fig. 23–24. If the index of refraction of the glass is 1.50, (a) what is the angle of refraction θ_A in the glass; (b) what is the angle θ_B at which the ray emerges from the glass?



Solution:

(a) $$(1.00)\sin 60.0° = (1.50)\sin\theta_A$$

$$\sin\theta_A = \frac{1.00}{1.50}\sin 60.0° = 0.5774,$$

$$\theta_A = 35.3°.$$

(*b*) Since the faces of the glass are parallel, the incident angle at the second surface is also θ_A (geometry), so $\sin\theta_A = 0.5774$. At this second interface, $n_1 = 1.50$ and $n_2 = 1.00$. Thus the ray re-enters the air at an angle θ_B given by

$$\sin\theta_B = \frac{1.50}{1.00}\sin\theta_A = 0.866,$$

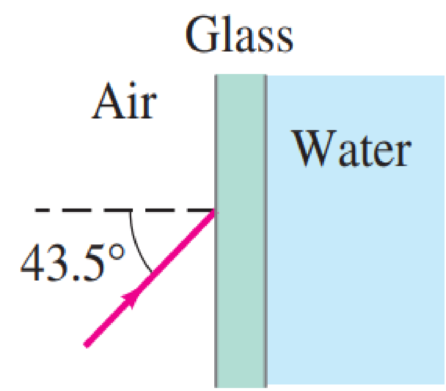
and $\theta_B = 60.0°$. The direction of a light ray is thus unchanged by passing through a flat piece of glass of uniform thickness.

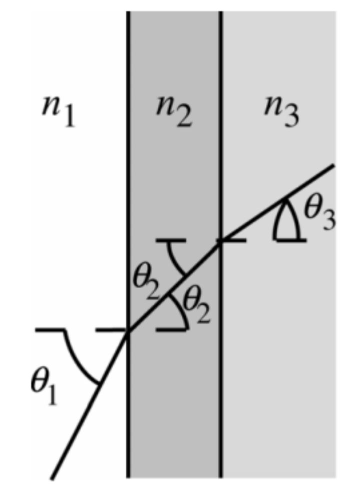
Problem 28:

A diver shines a flashlight upward from beneath the water at a 35.2° angle to the vertical. At what angle does the light leave the water?

Problem 31:

An aquarium filled with water has flat glass sides whose index of refraction is 1.54. A beam of light from outside the aquarium strikes the glass at a 43.5° angle to the perpendicular (Fig. 23–56). What is the angle of this light ray when it enters (a) the glass, and then (b) the water? (c) What would be the refracted angle if the ray entered the water directly?





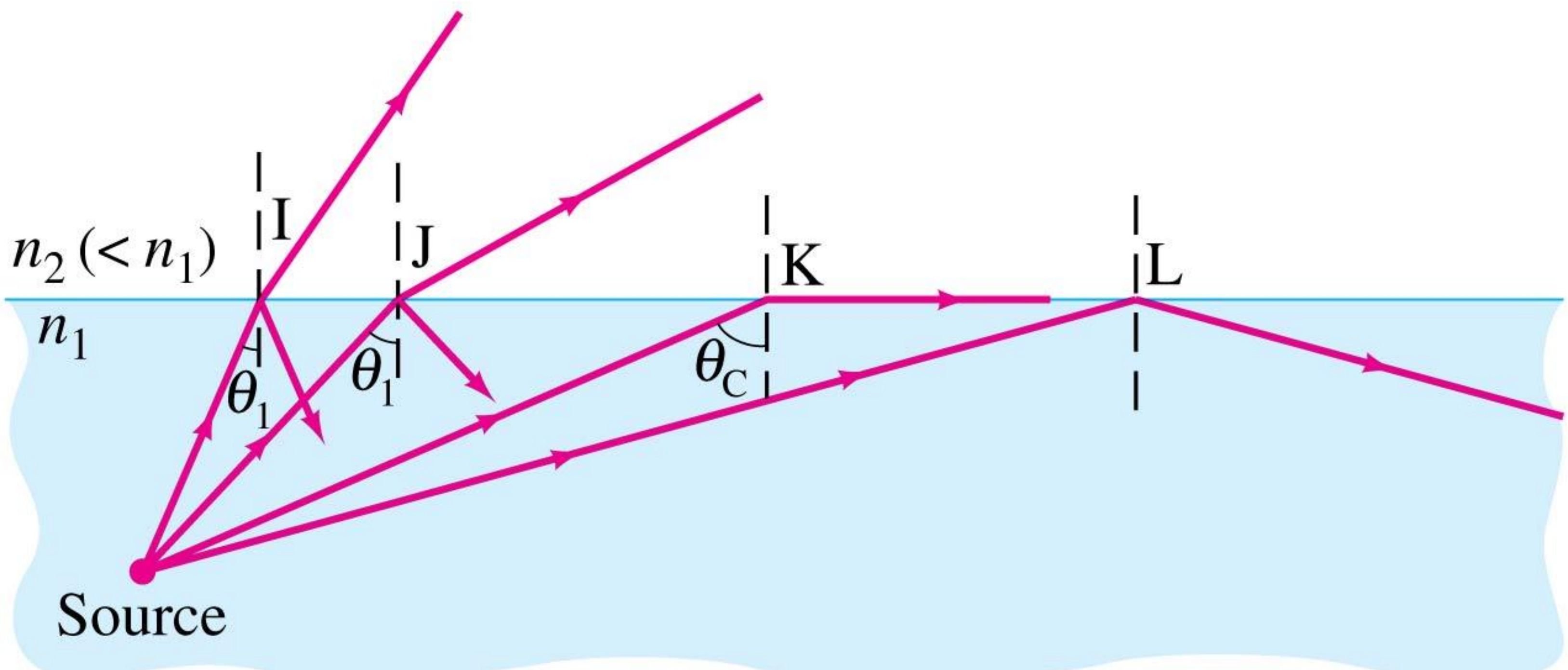
23-6 Total Internal Reflection; Fiber Optics

If light passes into a medium with a smaller index of refraction, the angle of refraction is larger. There is an angle of incidence for which the angle of refraction will be 90°; this is called the critical angle:

$$\sin\theta_C = \frac{n_2}{n_1}\sin 90° = \frac{n_2}{n_1}. \qquad (23\text{-}6)$$

23-6 Total Internal Reflection; Fiber Optics

If the angle of incidence is larger than this, no transmission occurs. This is called total internal reflection.



Example 23-10:

Describe what a person would see who looked up at the world from beneath the perfectly smooth surface of a lake or swimming pool.

Solution:

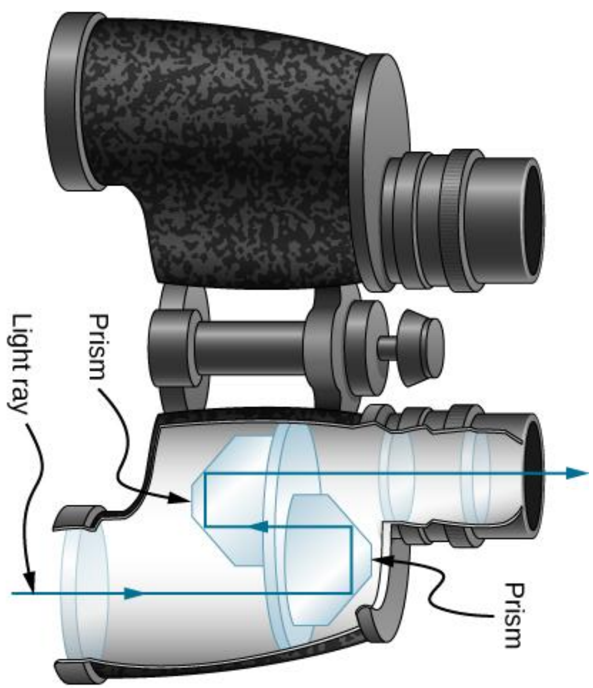
For an air–water interface, the critical angle is given by

$$\sin \theta_C = \frac{1.00}{1.33} = 0.750.$$

Therefore $\theta_C = 49°$. Thus the person would see the outside world compressed into a circle whose edge makes a 49° angle with the vertical. Beyond this angle, the person would see reflections from the sides and bottom of the lake or pool.

23-6 Total Internal Reflection; Fiber Optics

Binoculars often use total internal reflection; this gives true 100% reflection, which even the best mirror cannot do.



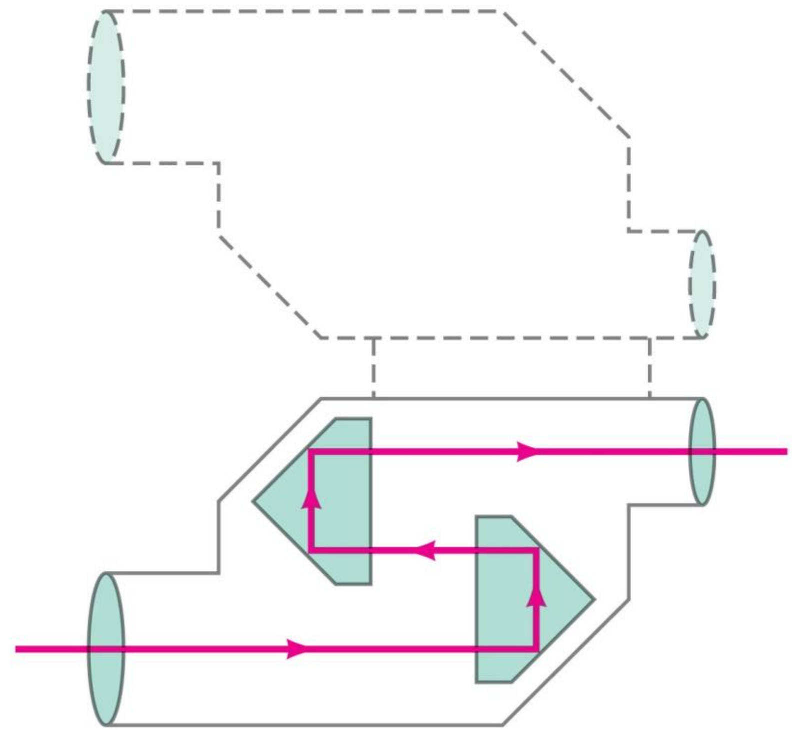
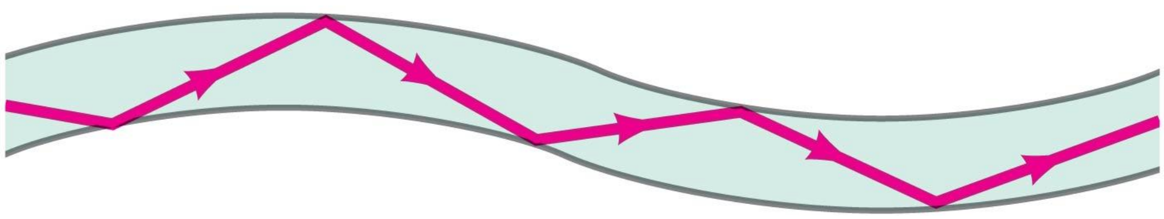
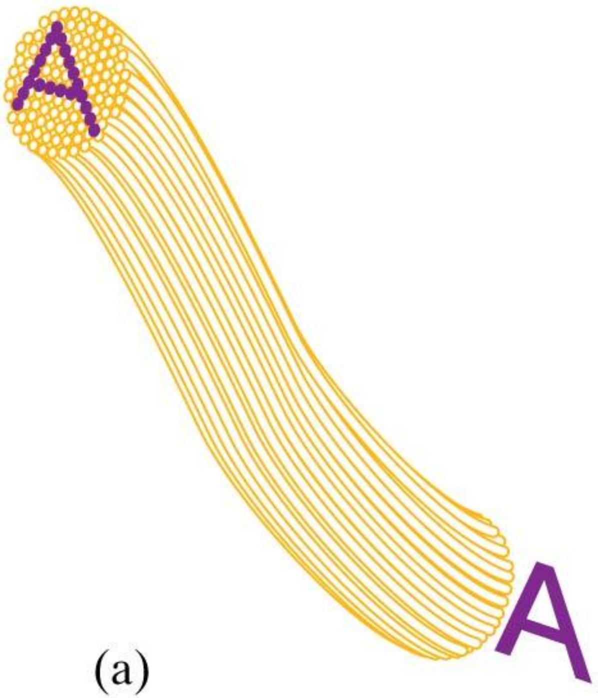


FIGURE 23–28 Total internal reflection of light by prisms in binoculars.

23-6 Total Internal Reflection; Fiber Optics

Total internal reflection is also the principle behind fiber optics. Light will be transmitted along the fiber even if it is not straight. An image can be formed using multiple small fibers.





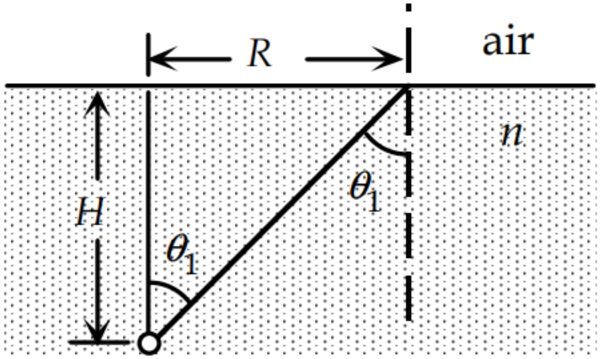
(a)

Problem 34:

The critical angle for a certain liquid–air surface is 47.2°. What is the index of refraction of the liquid?

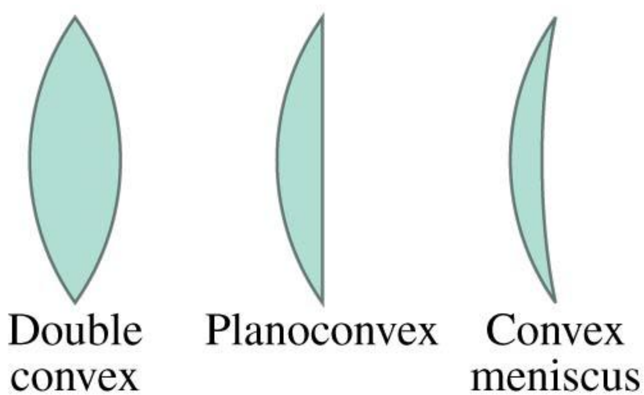
Problem 36:

A beam of light is emitted 8.0 cm beneath the surface of a liquid and strikes the air surface 7.6 cm from the point directly above the source. If total internal reflection occurs, what can you say about the index of refraction of the liquid?

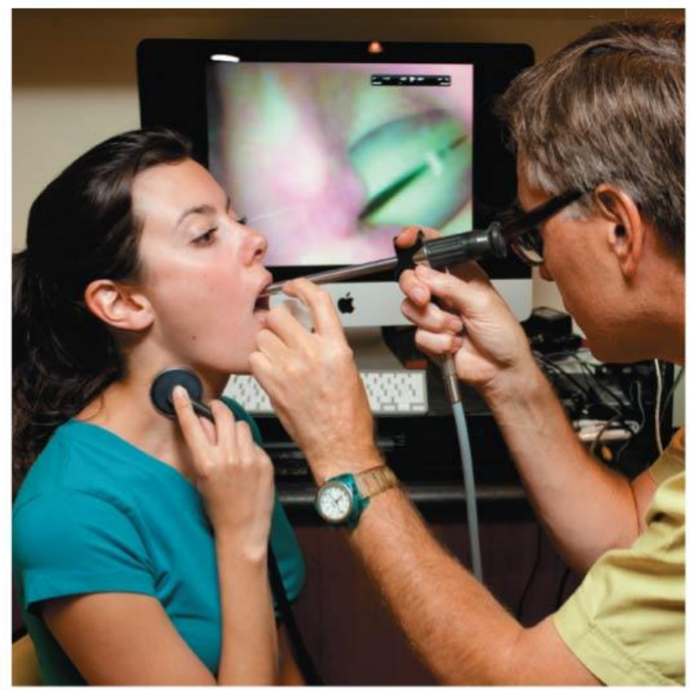


23-7 Thin Lenses; Ray Tracing

Thin lenses are those whose thickness is small compared to their radius of curvature. They may be either converging (a) or diverging (b).

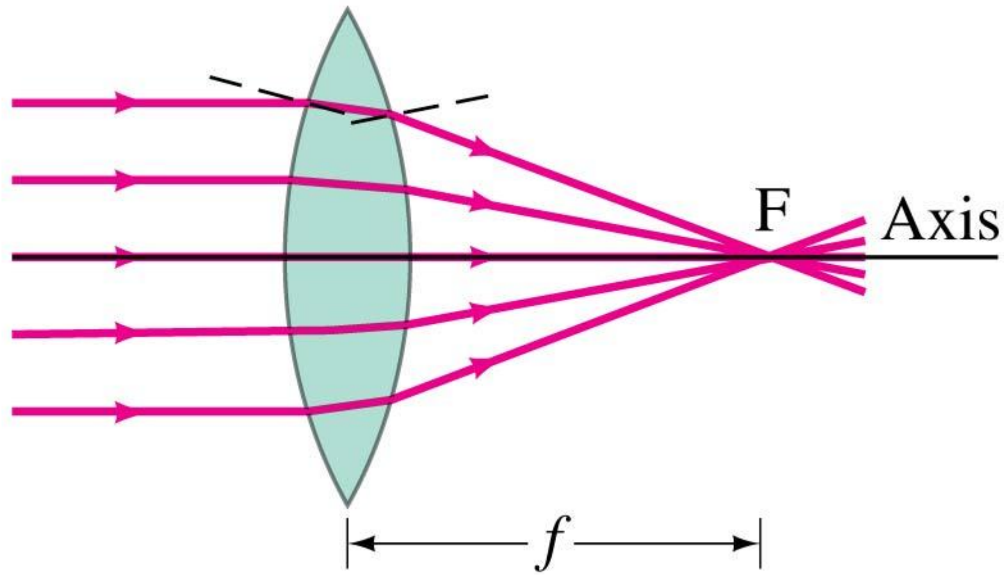


(a) Converging lenses


(b)

23-7 Thin Lenses; Ray Tracing

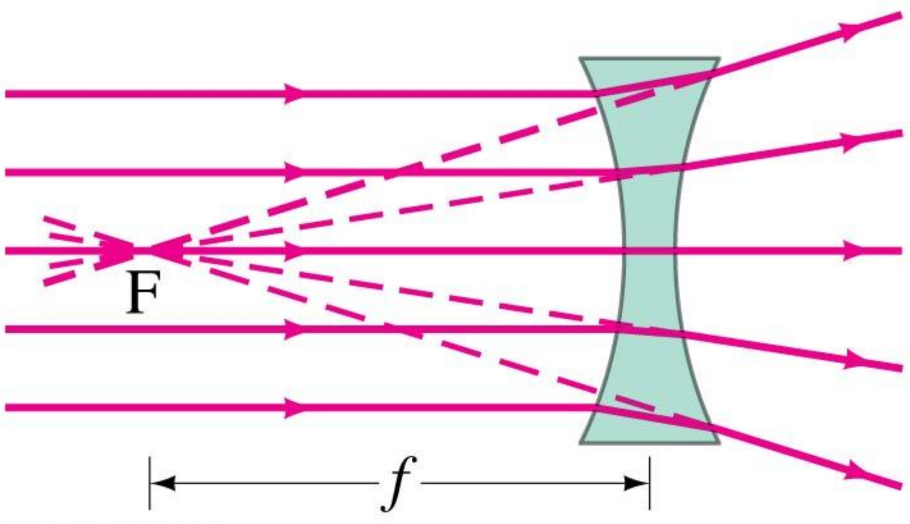
Parallel rays are brought to a focus by a converging lens (one that is thicker in the center than it is at the edge).



Convex **has a positive focal length**

23-7 Thin Lenses; Ray Tracing

A diverging lens (thicker at the edge than in the center) make parallel light diverge; the focal point is that point where the diverging rays would converge if projected back.



Concave **has a negative focal length**

23-7 Thin Lenses; Ray Tracing

The power of a lens is the inverse of its focal length.

$$P = \frac{1}{f}. \qquad (23\text{-}7)$$

Lens power is measured in diopters, D.

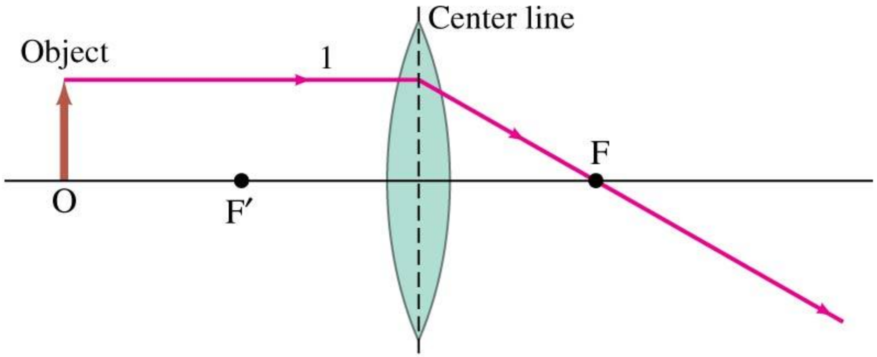
$$1\ \text{D} = 1\ \text{m}^{-1}$$

23-7 Thin Lenses; Ray Tracing

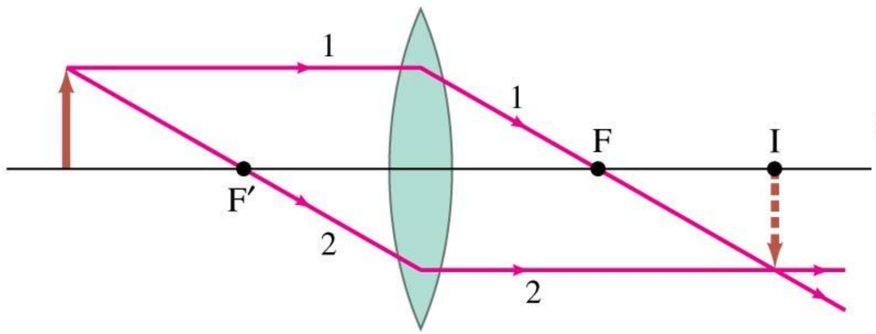
Ray tracing for thin lenses is similar to that for mirrors. We have three key rays:

1. This ray comes in parallel to the axis and exits through the focal point.
2. This ray comes in through the focal point and exits parallel to the axis.
3. This ray goes through the center of the lens and is undeflected.

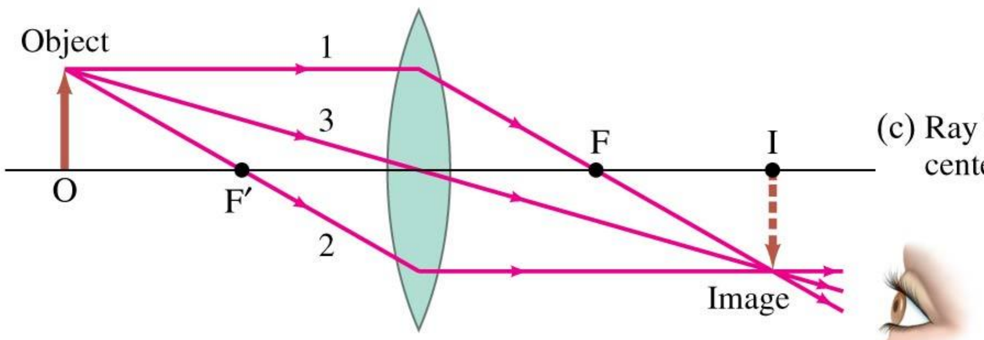
23-7 Thin Lenses; Ray Tracing



(a) Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



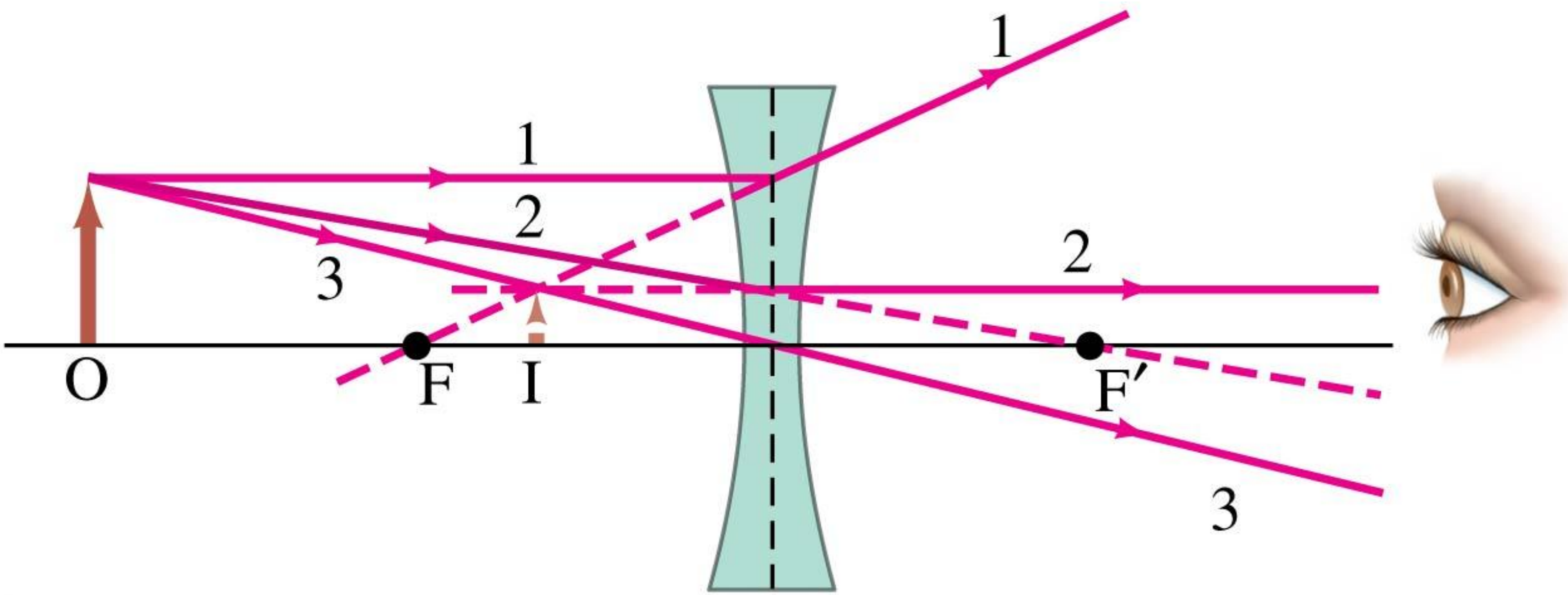
(b) Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.



(c) Ray 3 passes straight through the center of the lens (assumed very thin).

23-7 Thin Lenses; Ray Tracing

For a diverging lens, we can use the same three rays; the image is upright and virtual.



23-8 The Thin Lens Equation

The thin lens equation is the same as the mirror equation:

$$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$$

or

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \qquad (23\text{-}8)$$

23-8 The Thin Lens Equation

The sign conventions are slightly different:

1. The focal length is positive for converging lenses and negative for diverging.
2. The object distance is positive when the object is on the same side as the light entering the lens (not an issue except in compound systems); otherwise it is negative.
3. The image distance is positive if the image is on the opposite side from the light entering the lens; otherwise it is negative.
4. The height of the image is positive if the image is upright and negative otherwise.

23-8 The Thin Lens Equation

The magnification formula is also the same as that for a mirror:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad (23\text{-}9)$$

The power of a lens is positive if it is converging and negative if it is diverging.

23-8 The Thin Lens Equation

Problem Solving: Thin Lenses

1. Draw a ray diagram. The image is located where the key rays intersect.

2. Solve for unknowns.

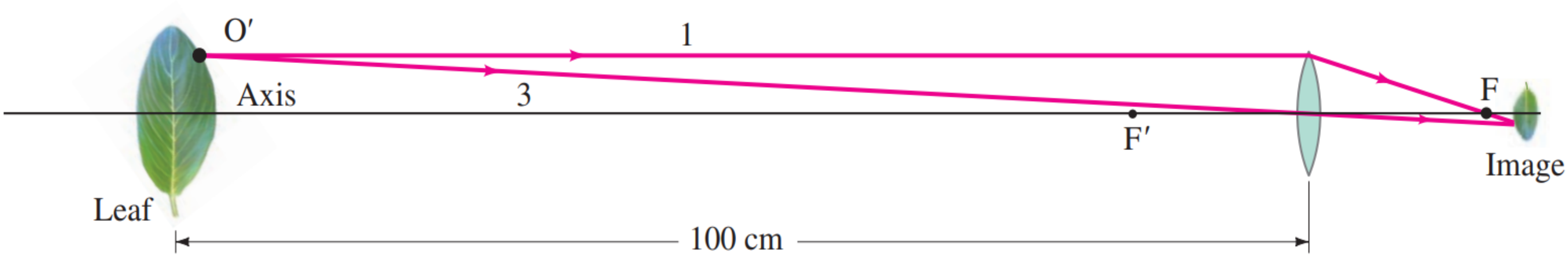
3. Follow the sign conventions.

4. Check that your answers are consistent with the ray diagram.

Example 23-12:

What is (a) the position, and (b) the size, of the image of a 7.6 cm high leaf placed 1.00 m from a +50.0 mm focal length camera lens?

Solution:



(a) $$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{5.00\text{ cm}} - \frac{1}{100\text{ cm}} = \frac{20.0 - 1.0}{100\text{ cm}} = \frac{19.0}{100\text{ cm}}.$$

$$d_i = \frac{100\text{ cm}}{19.0} = 5.26\text{ cm},$$ or 52.6 mm behind the lens.

(b) $$m = -\frac{d_i}{d_o} = -\frac{5.26\text{ cm}}{100\text{ cm}} = -0.0526,$$

$$h_i = mh_o = (-0.0526)(7.6\text{ cm}) = -0.40\text{ cm}.$$

The image is 4.0 mm high.

Sign conventions. The image distance came out positive, so the image is behind the lens. The image height is(-4.0 mm) ; the minus sign means the image is inverted.

Example 23-14:

Where must a small insect be placed if a 25 cm focal length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object?

Solution:

SOLUTION The lens is diverging, so f is negative: $f = -25\text{ cm}$. The image distance must be negative too because the image is in front of the lens (sign conventions), so $d_i = -20\text{ cm}$. The lens equation, Eq. 23–8, gives

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = -\frac{1}{25\text{ cm}} + \frac{1}{20\text{ cm}} = \frac{-4 + 5}{100\text{ cm}} = \frac{1}{100\text{ cm}}.$$

So the object must be 100 cm in front of the lens.

Problem 40:

Sunlight is observed to focus at a point 16.5 cm behind a lens. (a) What kind of lens is it? (b) What is its power in diopters?

Solution:

(*a*) Forming a real image from parallel rays requires a converging lens.

(*b*) We find the power of the lens from Eqs. 23–7 and 23–8. We treat the object distance as infinite.

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}=P=\frac{1}{\infty}+\frac{1}{0.165\text{ m}}=\boxed{6.06\text{ D}}$$

Problem 45:

A stamp collector uses a converging lens with focal length 28 cm to view a stamp 16 cm in front of the lens. (a)Where is the image located? (b) What is the magnification?

Problem 53:

How far apart are an object and an image formed by an 85 cm focal length converging lens if the image is 3.25X larger than the object and is real?

Summary of Chapter 23

- Light paths are called rays
- Index of refraction: $n = \frac{c}{v}.$
- Law of refraction (Snell's law):

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$

- Total internal reflection occurs when angle of incidence is greater than critical angle:

$$\sin\theta_C = \frac{n_2}{n_1}\sin 90° = \frac{n_2}{n_1}.$$

- A converging lens focuses incoming parallel rays to a point

Summary of Chapter 23

- A diverging lens spreads incoming rays so that they appear to come from a point
- Power of a lens: $P = \frac{1}{f}.$
- Thin lens equation: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$ or $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$
- Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$