CH10: Fluids

Department of Physics
The University of Jordan

Chapter 10: Fluids

- Three (common) phases of matter:

1. **Solid:** Maintains shape & size (approx.), even under large forces.
2. **Liquid:** No fixed shape. Takes shape of container.
3. **Gas:** Neither fixed shape, nor fixed volume. Expands to fill container.

Lump **2.** & **3.** into category of ***FLUIDS***.

Fluids: Have the ability to flow.

Sect. 10-2: Density & Specific Gravity

- ***Density,*** ρ (lower case Greek ***rho***, ***NOT*** **p**!) of object, mass **M** & volume **V**:

$$\rho \equiv (M/V) \qquad (kg/m^3 = 10^{-3}\ g/cm^3)$$

- ***Specific Gravity (SG):*** Ratio of density of a substance to density of water.

$$\rho_{water} = 1\ g/cm^3 = 1000\ kg/m^3$$

See table!!

$$\rho = (M/V) \quad SG = (\rho/\rho_{water}) = 10^{-3}\rho$$

$$(\rho_{water} = 10^3\ kg/m^3)$$

TABLE 10–1
Densities of Substances‡

Substance	Density, ρ (kg/m^3)
Solids	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3-0.9 \times 10^3$
Glass, common	$2.4-2.8 \times 10^3$
Ice (H_2O)	0.917×10^3
Bone	$1.7-2.0 \times 10^3$

TABLE 10–1
Densities of Substances‡

Substance	Density, ρ (kg/m^3)
Liquids	
Water (4°C)	1.000×10^3
Sea water	1.025×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	$0.7-0.8 \times 10^3$
Gases	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Water (steam) (100°C)	0.598

‡Densities are given at 0°C and 1 atm pressure unless otherwise specified.

- ***Note:*** $\rho = (\mathbf{M/V})$

$\Rightarrow$ **Mass** of body, density ρ, volume **V** is

$$\mathbf{M} = \rho \mathbf{V}$$

$\Rightarrow$ **Weight** of body, density ρ, volume **V** is

$$\mathbf{Mg} = \rho \mathbf{V g}$$

EXAMPLE 10 -1

What is the mass of a solid iron wrecking ball of radius 18 cm?

SOLUTION The volume of the sphere is

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}(3.14)(0.18\,\text{m})^3 = 0.024\,\text{m}^3.$$

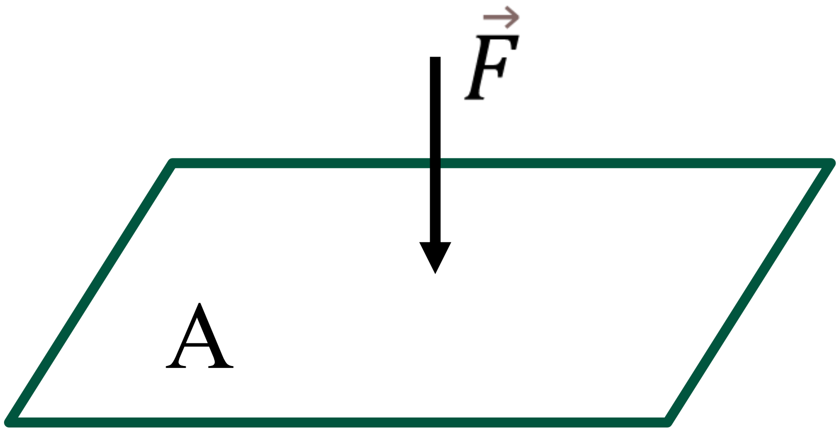
From Table 10–1, the density of iron is $\rho = 7800\,\text{kg/m}^3$, so Eq. 10–1 gives

$$m = \rho V = \left(7800\,\text{kg/m}^3\right)\left(0.024\,\text{m}^3\right) = 190\,\text{kg}.$$

Section 10-3: Pressure in Fluids

- Definition: Pressure = Force/Area

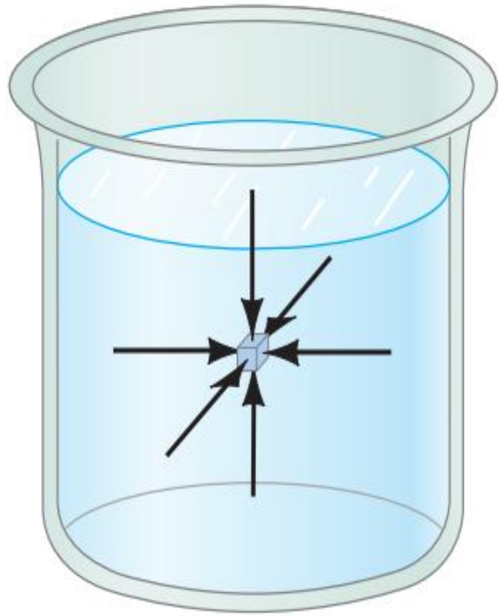
$$\mathbf{P \equiv (F/A)}$$



F applied **perpendicular** to **A**

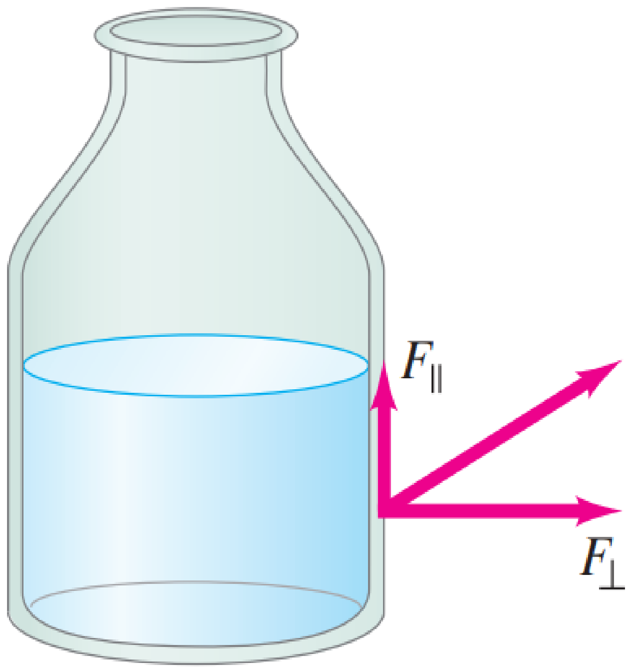
SI units: **N/m²**

$$\mathbf{1\ N/m^2 = 1\ Pa\ (Pascal)}$$



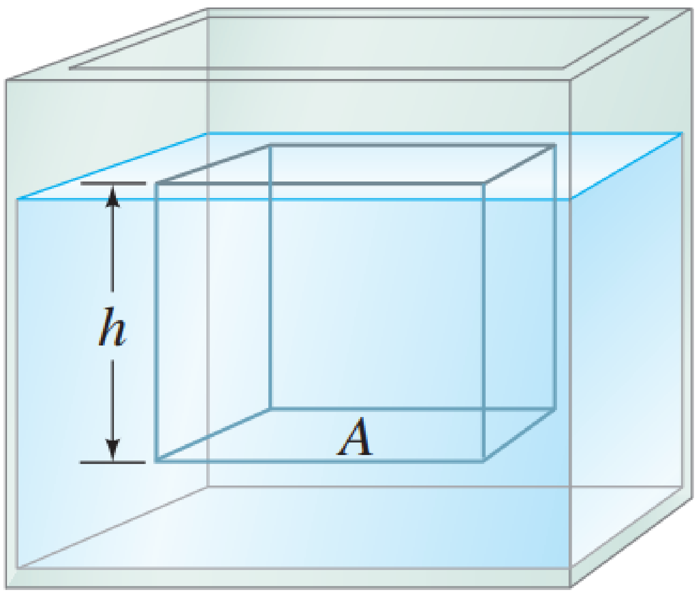
Pressure is the same in every direction in a nonmoving fluid at a given depth. If this weren’t true, the fluid would be in motion.

- **P** is ⊥ any fluid surface: $\mathbf{P} = (\mathbf{F}_{\perp} / \mathbf{A})$



If there were a component of force parallel to the solid surface of the container, the liquid would move in response to it. For a liquid at rest, $F_{\parallel} = 0$.

- ***Experimental Fact***: **Pressure depends on depth.** Consider a fluid ***at rest***. Depth **h** below surface:



Calculating the pressure at a depth h in a liquid, due to the weight of the liquid above.

At rest $\Rightarrow \sum \mathbf{F_y} = \mathbf{0}$

or, $\mathbf{F - mg = 0 \Rightarrow F = mg}$

$\mathbf{F = mg} = \rho \mathbf{Vg,\ V = Ah}$

$\Rightarrow \mathbf{F} = \rho \mathbf{Ahg}$

$\Rightarrow \mathbf{P \equiv F/A} = \rho \mathbf{gh}$

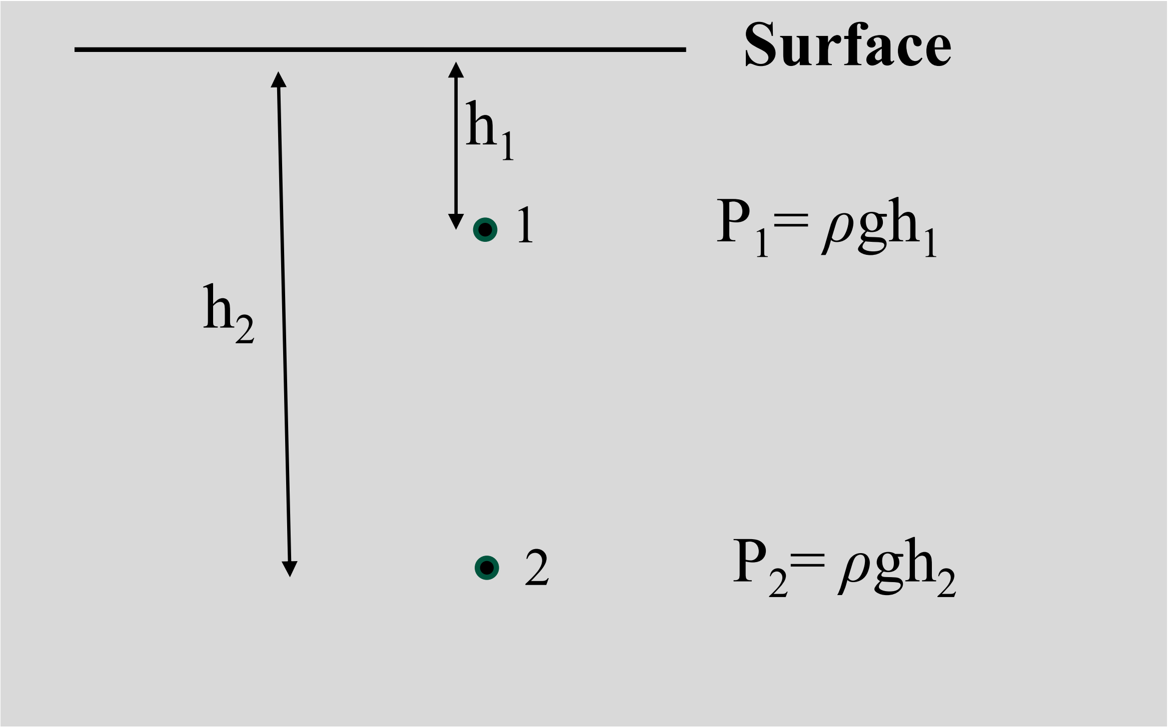
Pressure at depth **h** (fluid at rest)

$$\mathbf{P} = \rho \mathbf{gh}$$

- Depth **h** below surface of liquid: $\mathbf{P = \rho g h}$

⇒ ***Change in pressure with change in depth***:

$$\Delta P = \rho g \Delta h$$ (for a fluid ***at rest*** only!)



$$\Delta P = P_2 - P_1$$
$$= \rho g (h_2 - h_1)$$
$$\rho g \Delta h$$

EXAMPLE 10 -2

A 60-kg person's two feet cover an area of 500 cm^2.
(a) Determine the pressure exerted by the two feet on the ground. (b) If the person stands on one foot, what will be the pressure under that foot?

SOLUTION (*a*) The pressure on the ground exerted by the two feet is

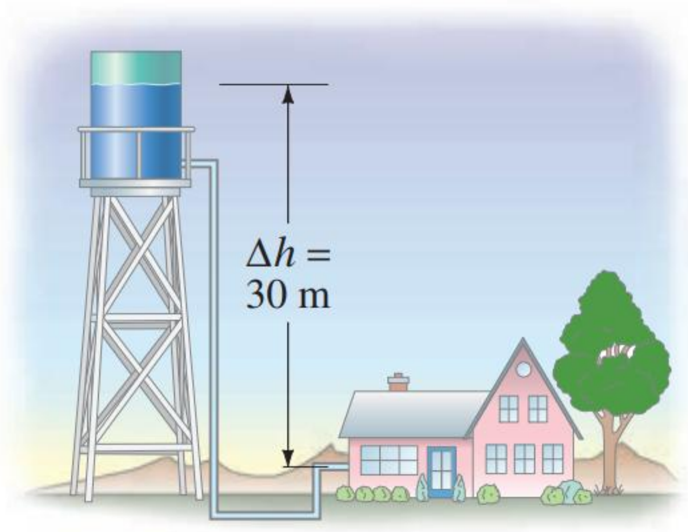
$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(60\,\text{kg})(9.8\,\text{m/s}^2)}{(0.050\,\text{m}^2)} = 12 \times 10^3\,\text{N/m}^2.$$

(*b*) If the person stands on one foot, the force is still equal to the person's weight, but the area will be half as much, so the pressure will be twice as much: $24 \times 10^3\,\text{N/m}^2$.

EXAMPLE 10 -3

The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house, Fig. 10–4. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

FIGURE 10–4 Example 10–3.



SOLUTION We assume the atmospheric pressure at the surface of the water in the storage tank is the same as at the faucet. So, the water pressure difference between the faucet and the surface of the water in the tank is

$$\Delta P = \rho g \Delta h = (1.0 \times 10^3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(30\,\mathrm{m}) = 2.9 \times 10^5\,\mathrm{N/m^2}.$$

Section 10-4: Atmospheric Pressure

- **Earth's atmosphere:** A fluid.
  - But doesn't have a fixed top "surface"!
- Change in height **Δh** above Earth's surface:

⇒ Change in pressure: $\Delta P = \rho g \Delta h$

- Sea level: $P_0 \equiv 1.013 \times 10^5\ N/m^2$

  $= 101.3\ kPa \equiv 1\ atm$

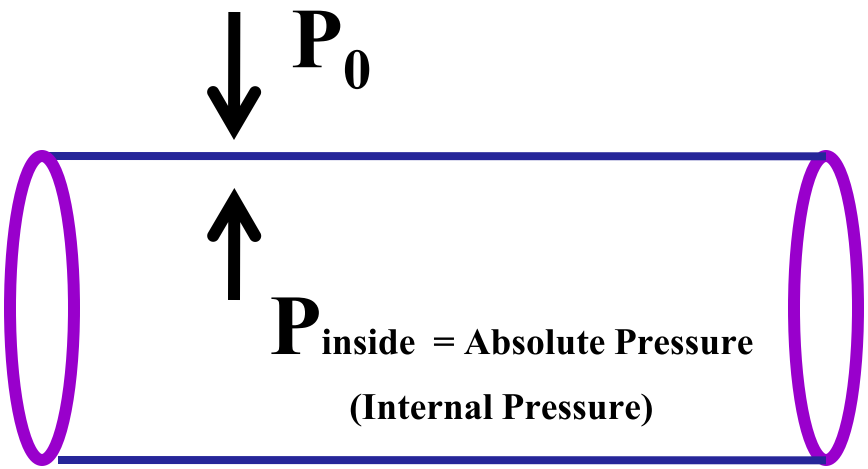
  - Old units: **1 bar = 1.00 × 10⁵ N/m²**

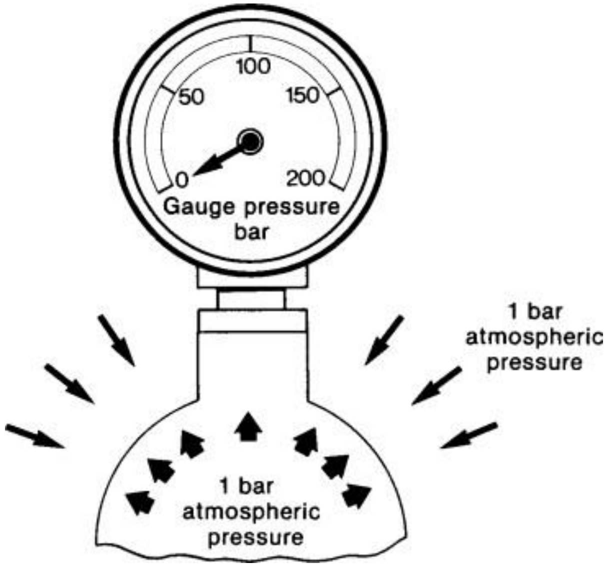
  → $1\ bar = 1.00 \times 10^5\ N/m^2$
- ***Physics:*** **Cause** of pressure at any height:

  Weight of air above that height!

Gauge Pressure

- Pressure gauges (like tire gauges, etc.) measure difference between atmospheric pressure $\mathbf{P_0}$ & internal pressure (of tire, for example).
- **Gauge pressure:** $\mathbf{P_G = P - P_0}$





Gauge Pressure

It is important to note that tire gauges, and most other pressure gauges, register the pressure above and beyond atmospheric pressure. This is called gauge pressure. Thus, to get the absolute pressure, P, we must add the atmospheric pressure, to the gauge pressure, P_0, to the gauge pressure, P_G:

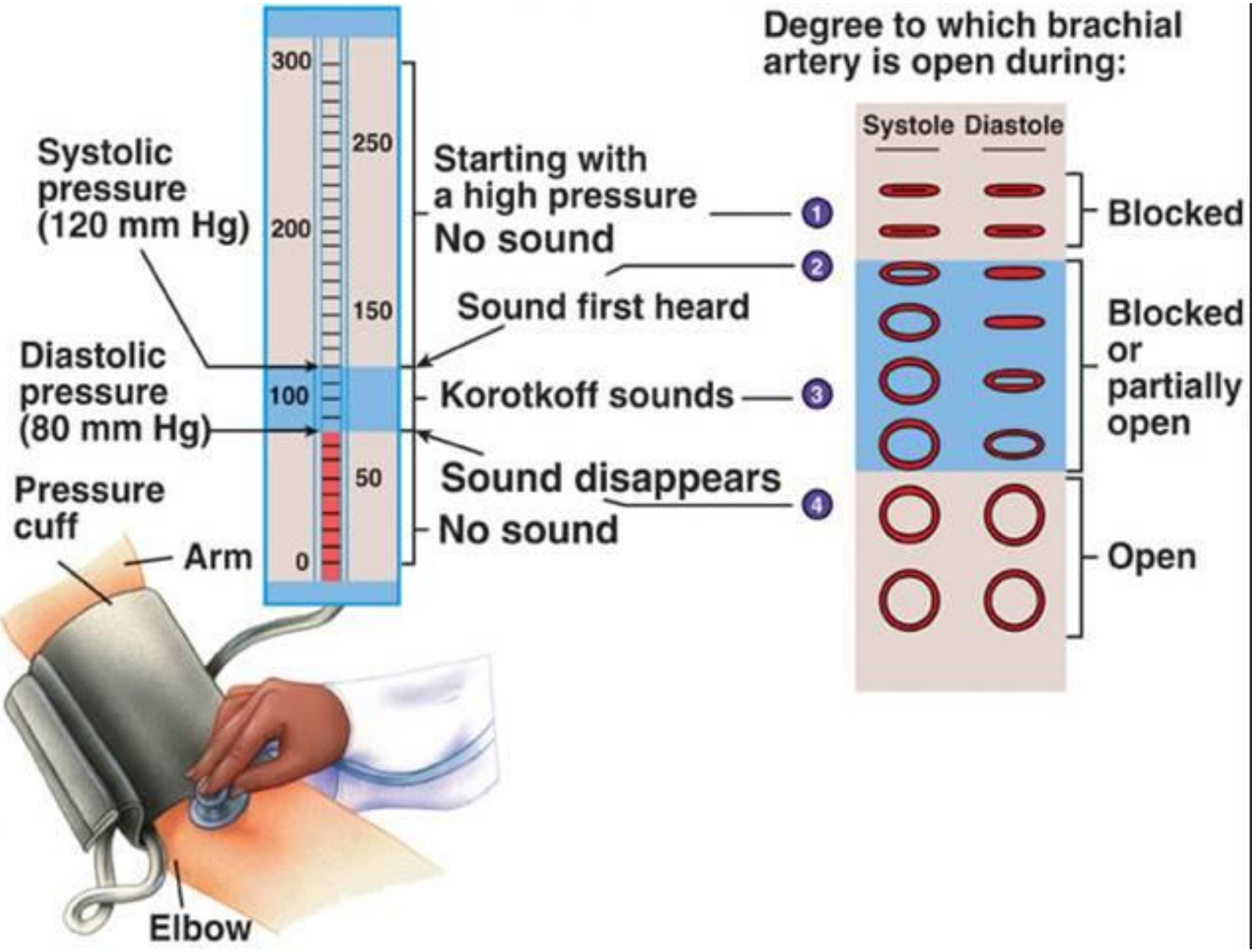
$$P = P_G + P_0$$

If a tire gauge registers 220 kPa, the absolute pressure within the tire is 220 kPa + 101 kPa = 321 kPa equivalent to about 3.2 atm (2.2 atm gauge pressure).

The Blood Pressure is a Gauge Pressure

This is quite obvious when you look at numerical values. A normal blood pressure is about 120 mmHg . A typical atmospheric pressure is about 760 mmHg. Since the blood pressure is lower than the atmospheric pressure, it can only be a gauge pressure. The corresponding absolute pressure would be about 880 mmHg.

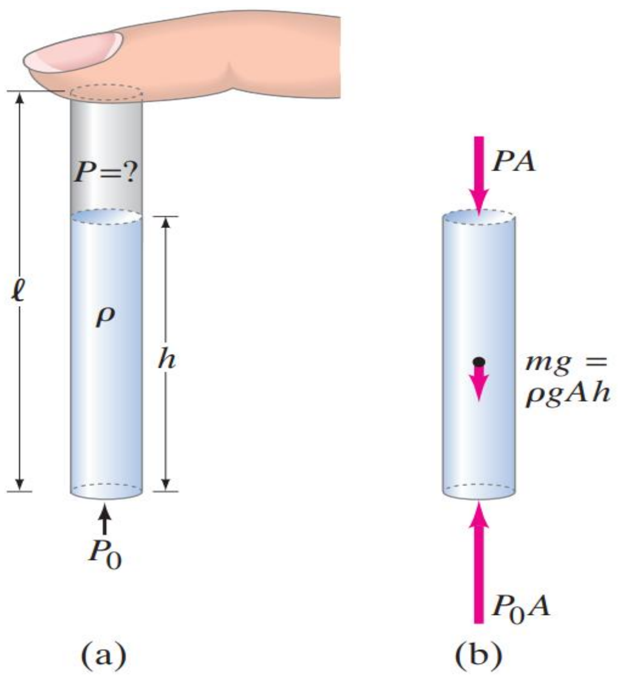
The Normal Blood Pressure is 120/80. mmHg



EXAMPLE 10 -4

You insert a straw of length into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water (Fig. 10–5a). Does the air in the space between your finger and the top of the water have a pressure P that is greater than, equal to, or less than, the atmospheric pressure outside the straw?

RESPONSE Consider the forces on the column of water (Fig. 10–5b). Atmospheric pressure outside the straw pushes upward on the water at the bottom of the straw, gravity pulls the water downward, and the air pressure inside the top of the straw pushes downward on the water. Since the water is in equilibrium, the upward force due to atmospheric pressure must balance the two downward forces. The only way this is possible is for the air pressure P inside the straw at the top to be less than the atmosphere pressure outside the straw. (When you initially remove the straw from the water glass, a little water may leave the bottom of the straw, thus increasing the volume of trapped air and reducing its density and pressure.)



Section 10-5: Pascal’s Principle

States that: *if an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.* ≡ ***Pascal’s Principle***

French scientist Blaise Pascal (1623–1662).

- Simple example: Water in a lake (at rest). At depth h below surface, pressure is

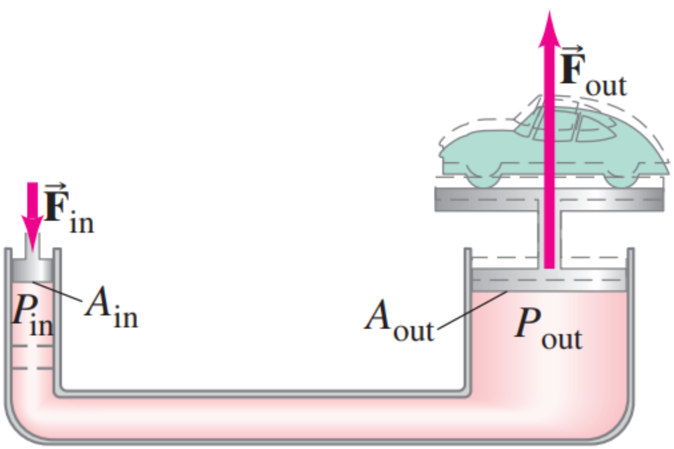
  $\mathbf{P = P_0 + \rho g h}$ ($\mathbf{P_0}$ = atmospheric pressure)

Pascal's Principle

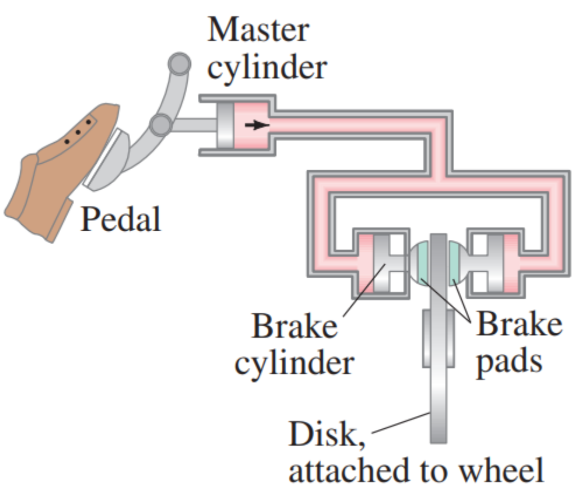
The input quantities are represented by the subscript "in" and the output by "out."

$$P_{\text{out}} = P_{\text{in}} \quad \Rightarrow \quad \frac{F_{\text{out}}}{A_{\text{out}}} = \frac{F_{\text{in}}}{A_{\text{in}}},$$

$$\frac{F_{\text{out}}}{F_{\text{in}}} = \frac{A_{\text{out}}}{A_{\text{in}}}$$



hydraulic lift



hydraulic brakes in a car

Section 10-6: Pressure Measurement

- Many types of pressure measurement devices. Most use $\mathbf{P - P_0 = \rho g \Delta h = P_G}$ = gauge pressure

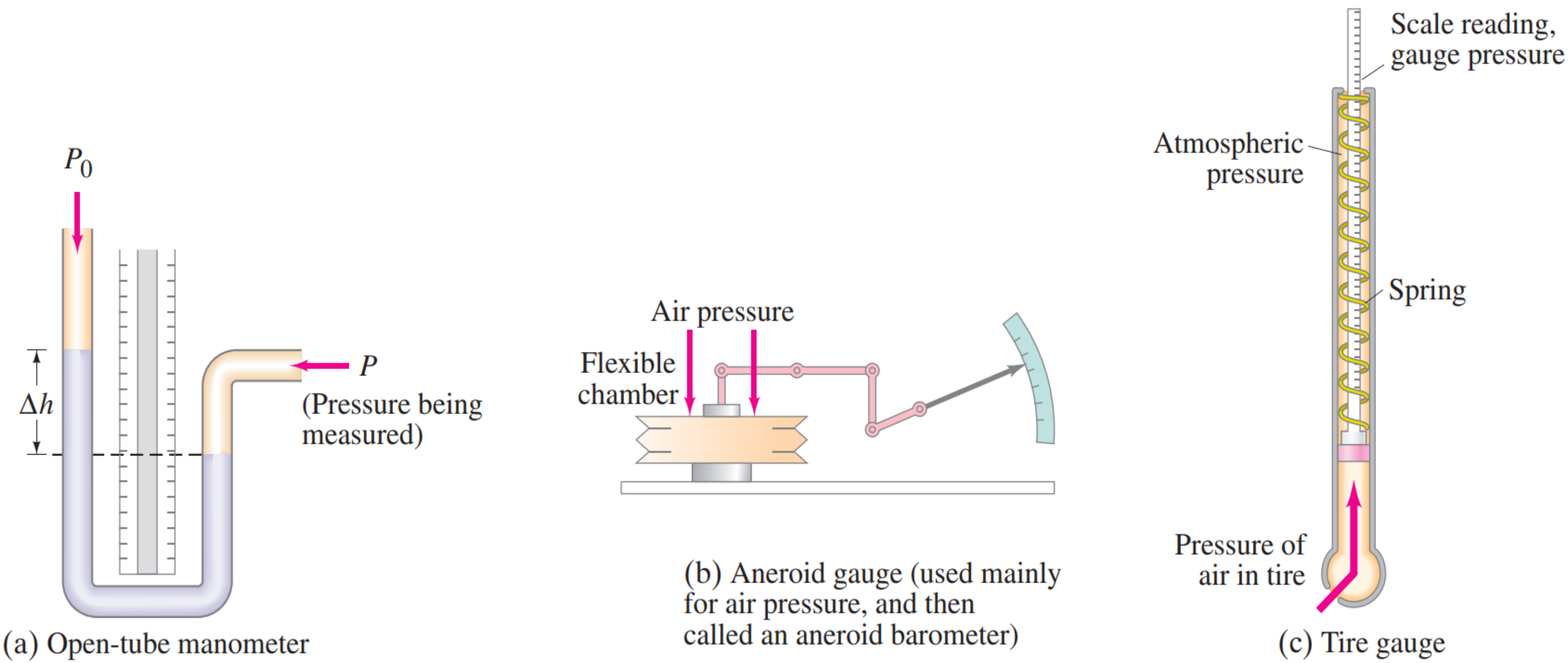


FIGURE 10–7 Pressure gauges: (a) open-tube manometer, (b) aneroid gauge, and (c) common tire pressure gauge.

Various Pressure Units

- Gauge Pressure: $\mathbf{P_A} = \boldsymbol{\rho}\mathbf{gh}$

⇒ ***Alternate unit of pressure:*** Instead of calculating $\boldsymbol{\rho}\mathbf{gh}$, common to use standard liquid (mercury, Hg or alcohol, where $\boldsymbol{\rho}$ is standard) & measure **h**

⇒ **Quote pressure in length units**! For example:

"millimeters of mercury" ≡ **mm Hg**

For **h = 1 mm Hg = $\mathbf{10^{-3}}$ m Hg**

$$\rho_{mercury}\, gh = (1.36 \times 10^4\ kg/m^3)\ (9.8\ m/s^2)(10^{-3}\ m)$$

$$= 133\ N/m^2 = 133\ Pa \equiv 1\ Torr$$

(another pressure unit!)

mm Hg & **Torr** are not proper SI pressure units!

- About as many ***pressure units*** as there are measurement devices!!

TABLE 10–2 Conversion Factors Between Different Units of Pressure

In Terms of $1\ Pa = 1\ N/m^2$	1 atm in Different Units
$1\ atm = 1.013 \times 10^5\ N/m^2$	$1\ atm = 1.013 \times 10^5\ N/m^2$
$= 1.013 \times 10^5\ Pa = 101.3\ kPa$	
$1\ bar = 1.000 \times 10^5\ N/m^2$	$1\ atm = 1.013\ bar$
$1\ dyne/cm^2 = 0.1\ N/m^2$	$1\ atm = 1.013 \times 10^6\ dyne/cm^2$
$1\ lb/in.^2 = 6.90 \times 10^3\ N/m^2$	$1\ atm = 14.7\ lb/in.^2$
$1\ lb/ft^2 = 47.9\ N/m^2$	$1\ atm = 2.12 \times 10^3\ lb/ft^2$
$1\ cm\text{-}Hg = 1.33 \times 10^3\ N/m^2$	$1\ atm = 76.0\ cm\text{-}Hg$
$1\ mm\text{-}Hg = 133\ N/m^2$	$1\ atm = 760\ mm\text{-}Hg$
$1\ torr = 133\ N/m^2$	$1\ atm = 760\ torr$
$1\ mm\text{-}H_2O\ (4°C) = 9.80\ N/m^2$	$1\ atm = 1.03 \times 10^4\ mm\text{-}H_2O\ (4°C)$
	$\approx 10\ m$ of water

- ***Preferred (SI)*** unit: **1 Pa** (Pascal) **= 1 N/m²**

Mercury Barometer

- **Weather reports:** Barometric pressure (atmospheric pressure): **28-32 inches Hg**

  76 cm = 760 mm

  = 29.29 inches

  When **h = 760 mm,**

  $\mathbf{P} = \rho_{\mathbf{mercury}}\,\mathbf{gh} = (13.6 \times 10^3\,\mathrm{kg/m^3})(9.80\,\mathrm{m/s^2})(0.760\,\mathrm{m})$

  $= \mathbf{1.013 \times 10^5\ N/m^2}$

  = 1 atm

- If use water

  $\mathbf{P = 1atm} = \rho_{\mathbf{water}}\,\mathbf{gh}$

$\Rightarrow$ **h ≈ 10 m ≈ 30 feet!**

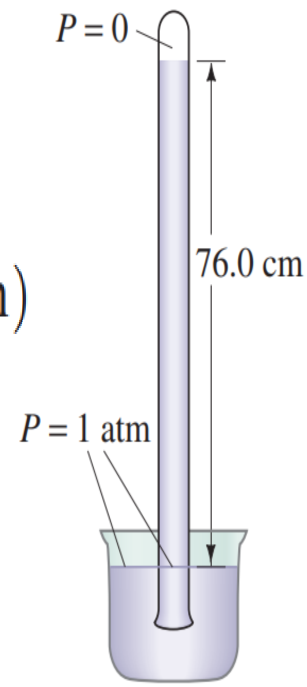


FIGURE 10–8 A mercury barometer, invented by Torricelli, is shown here when the air pressure is standard atmospheric, 76.0 cm-Hg.

Problem 17:

A house at the bottom of a hill is fed by a full tank of water 6.0 m deep and connected to the house by a pipe that is 75 m long at an angle of 61° from the horizontal (Fig. 10–49). (a) Determine the water gauge pressure at the house. (b) How high could the water shoot if it came vertically out of a broken pipe in front of the house?

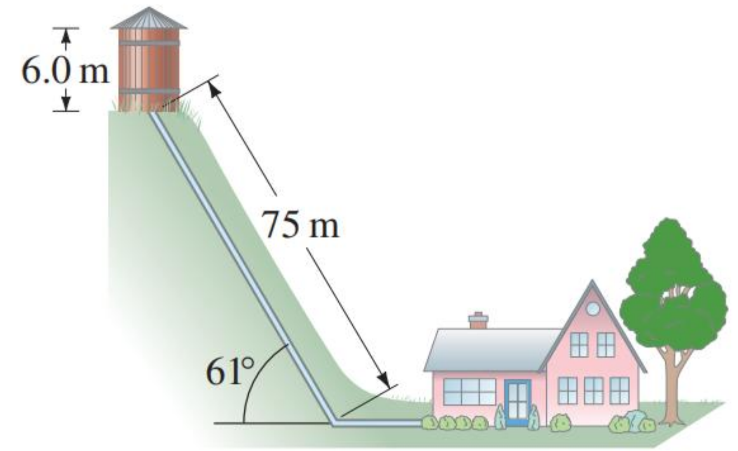
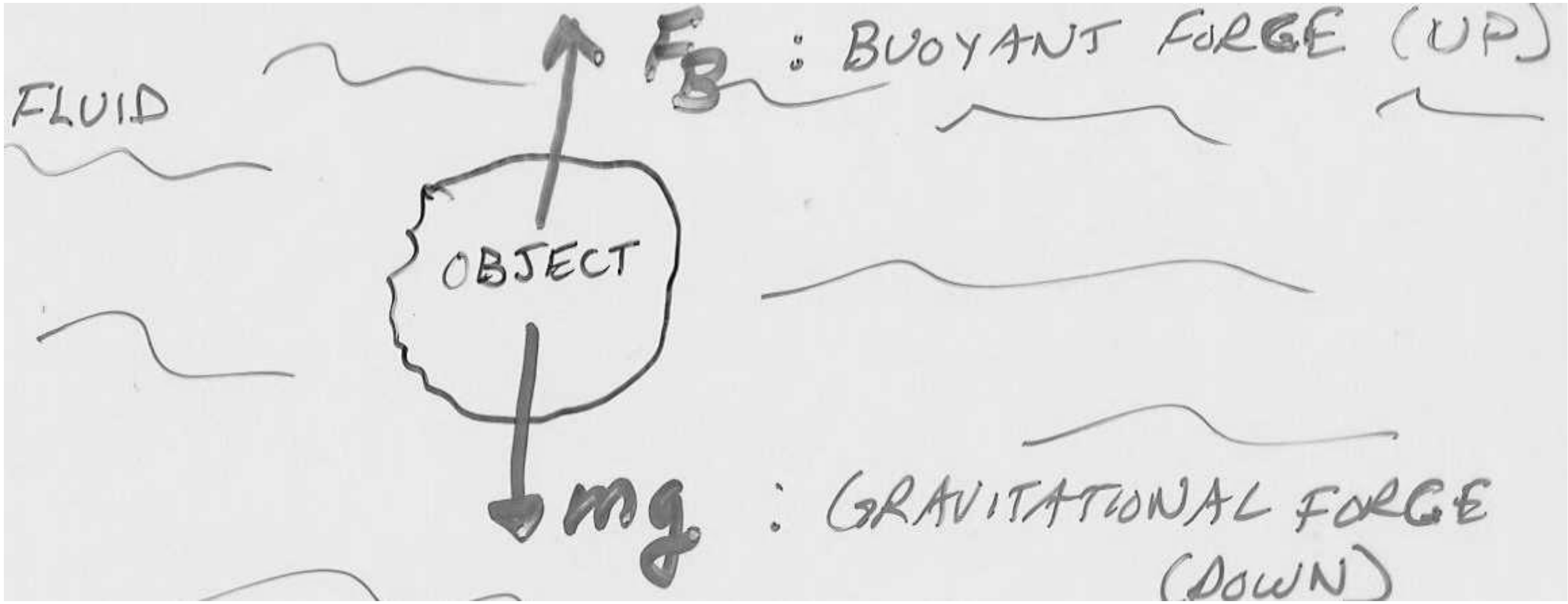


FIGURE 10–49 Problem 17.

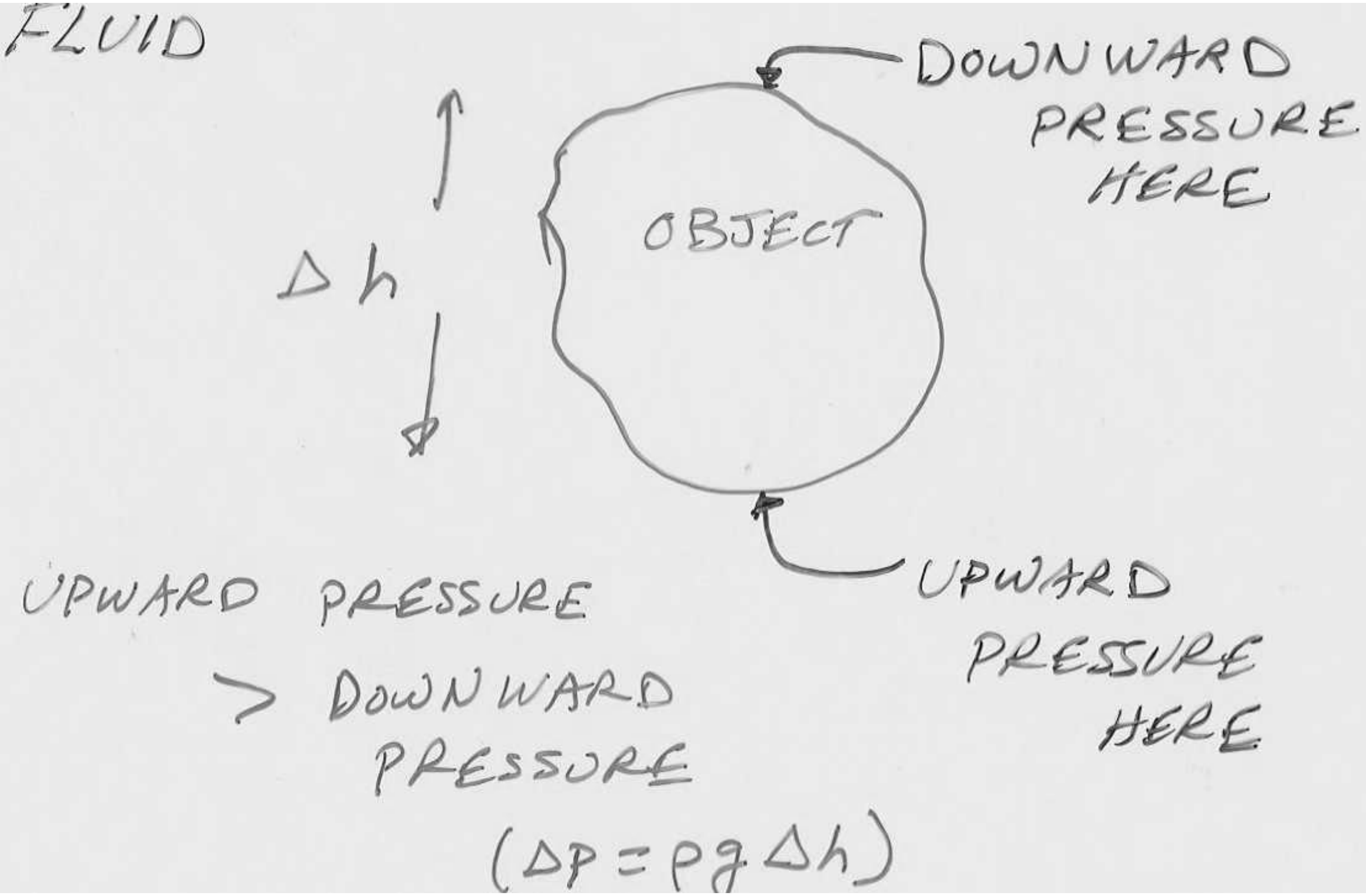
Sect. 10-7: Buoyancy/Archimedes Principle

- ***Experimental facts:***
  - Objects submerged (or partially submerged) in a fluid **APPEAR** to "weigh" less than in air.
  - When placed in a fluid, many objects float!
  - Both are examples of ***BUOYANCY.***



- ***Buoyant Force:*** Occurs ***because*** the pressure in a fluid increases with depth!

$$\Delta P = \rho g \, \Delta h$$ (fluid at ***REST!!***)

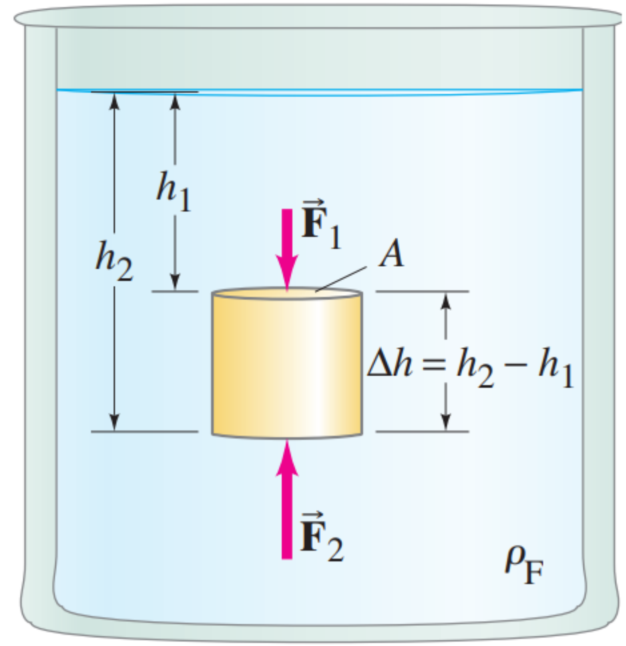


- Force due to pressure on top of the cylinder

$$F_1 = P_1 A = \rho_F g h_1 A$$

- Force due to pressure on bottom of the cylinder

$$F_2 = P_2 A = \rho_F g h_2 A$$



Cylinder of height Δh whose top and bottom ends have an area A and which is completely submerged in a fluid of density ρ_F

- The net force on the cylinder exerted by the fluid pressure, which is the buoyant force $\overrightarrow{F_B}$, acts upward and has the magnitude:

$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

$V = A \Delta h$ is the volume of the cylinder

$\rho_F V$ is mass of the fluid displaced

$\rho_F V g = m_F g$

weight of fluid which takes up a volume equal to the volume of the cylinder.

Archimedes Principle

⇒ The total (upward) buoyant force $\mathbf{F_B}$ on an **object** of volume **V** completely or partially submerged in a **fluid** with density ρ_F:

$$F_B = \rho_F V_{displ}\, g \qquad (1)$$

$V_{displ} \equiv$ Displaced Volume.

$V_{displ} \equiv V_{object} = V$ for submerged object.

$\rho_F V\, g \equiv m_F\, g \equiv$ weight of displaced fluid.

⇒ ***Upward buoyant force***

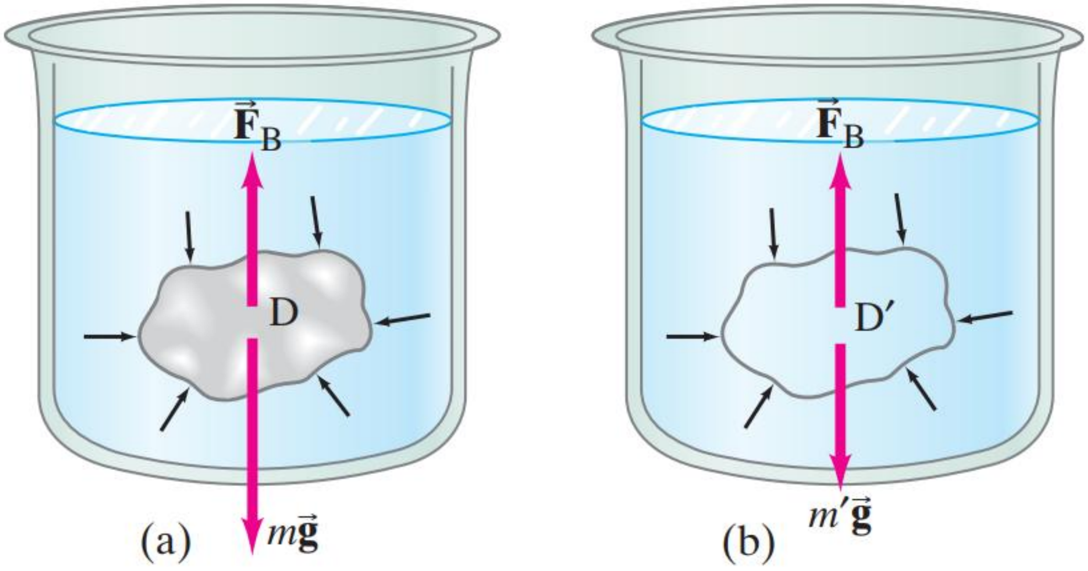
$$F_B = m_F g \qquad (2)$$

$F_B =$ weight of fluid displaced by the object!

(1) or **(2)** ≡ ***Archimedes Principle***

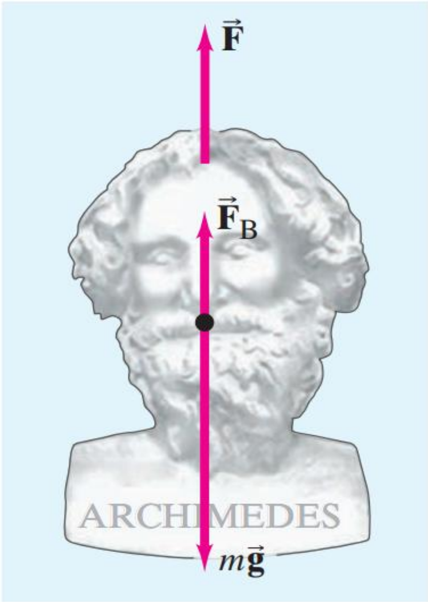
Proved for cylinder. Can show valid for any shape

Archimedes Principle



EXAMPLE 10 -7

A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4\ cm^3$. How much force is needed to lift it (without acceleration)?



Solution:

$$\Sigma F = ma = 0, \qquad F + F_B - mg = 0 \qquad F = mg - F_B$$

$$mg = (70\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 6.9 \times 10^2\,\mathrm{N}.$$

$$F_B = m_{H_2O}\,g = \rho_{H_2O} V g = (1.025 \times 10^3\,\mathrm{kg/m^3})(3.0 \times 10^{-2}\,\mathrm{m^3})(9.8\,\mathrm{m/s^2}) = 3.0 \times 10^2\,\mathrm{N},$$

F = 690 N – 300 N = 390 N

It is as if the statue had a mass of only $(390\,\mathrm{N})/(9.8\,\mathrm{m/s^2}) = 40\,\mathrm{kg}$.

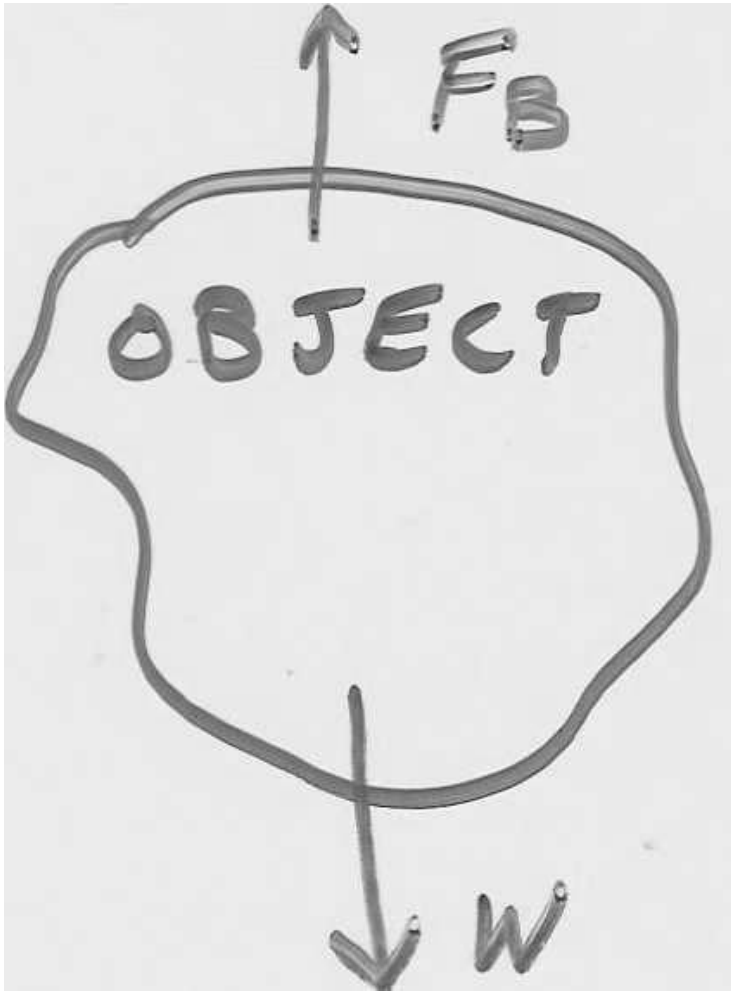
NOTE Here $F = 390\,\mathrm{N}$ is the force needed to lift the statue without acceleration when it is under water. As the statue comes *out* of the water, the force F increases, reaching 690 N when the statue is fully out of the water.

- Object, mass **m** in a fluid. Vertical forces are buoyant force, $\mathbf{F_B}$ & weight, $\mathbf{W = mg}$
- "Apparent weight"

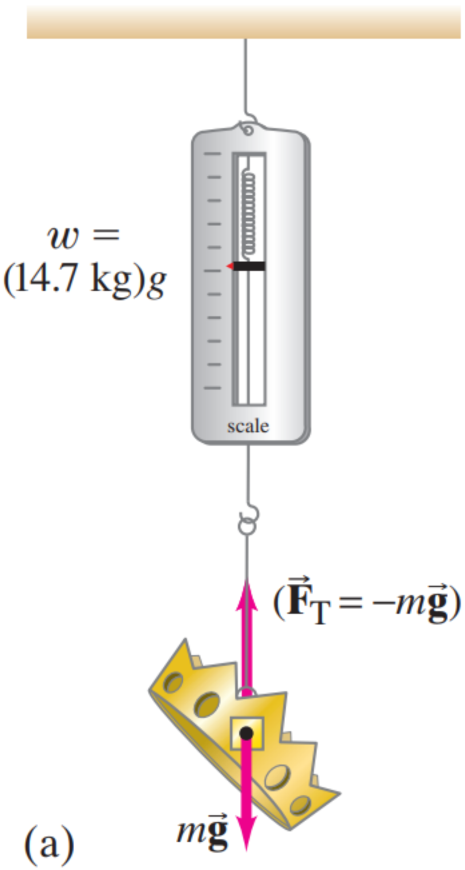
  = net downward force:

$$\mathbf{W' \equiv \sum F_y = W - F_B < W}$$

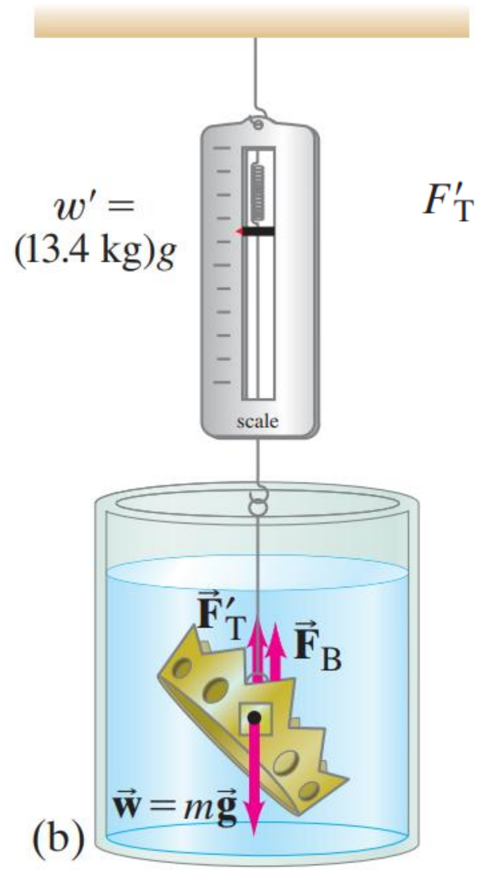
⇒ **Object appears "lighter"!**



Archimedes Principle & “Bath Legend”



$F_T = mg$



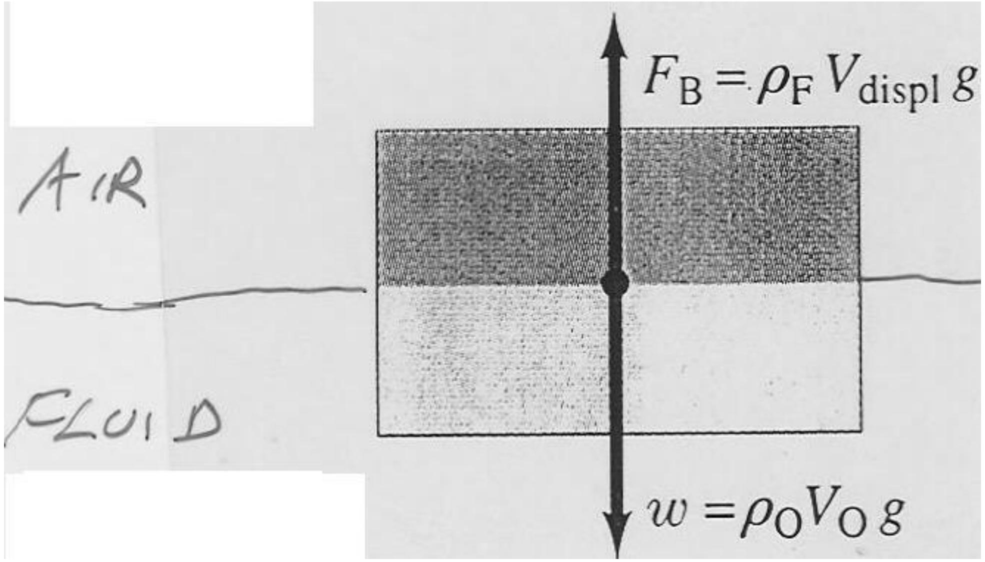
$F'_T + F_B = mg \; (= w)$

The scale now reads $m' = 13.4$ kg, where m' is related to the effective weight by $w' = m'g$. Thus
$F'_T = w' = w - F_B$.

- **Archimedes Principle:** Valid for floating objects

$$\mathbf{F_B = m_F g}$$
$$\mathbf{= \rho_F V_{displ}\, g}$$

($\mathbf{m_F}$ = mass of fluid displaced, $\mathbf{V_{displ}}$ = volume displaced)



$$\mathbf{W = m_O g = \rho_O V_O g}$$

($\mathbf{m_O}$ = mass of object, $\mathbf{V_O}$ = volume of object)

FIGURE 10–14 An object floating in equilibrium: $F_B = w$.

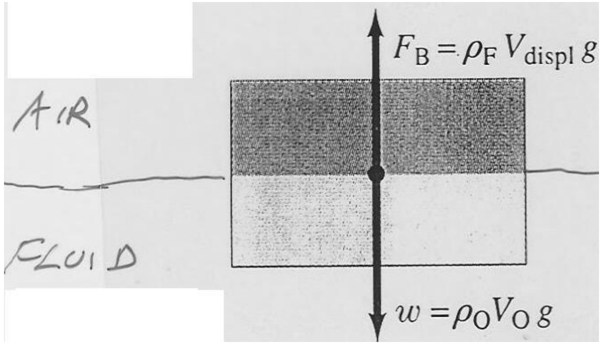
Equilibrium: $\Rightarrow$ $\sum \mathbf{F_y = 0 = F_B - W}$ $\Rightarrow$

- **Archimedes Principle**: Floating objects

Equilibrium: $\Rightarrow$ $\sum F_y = 0 = F_B - W$ $\Rightarrow$

$$F_B = W$$

or $\rho_F V_{displ}\, g = \rho_O V_O g$

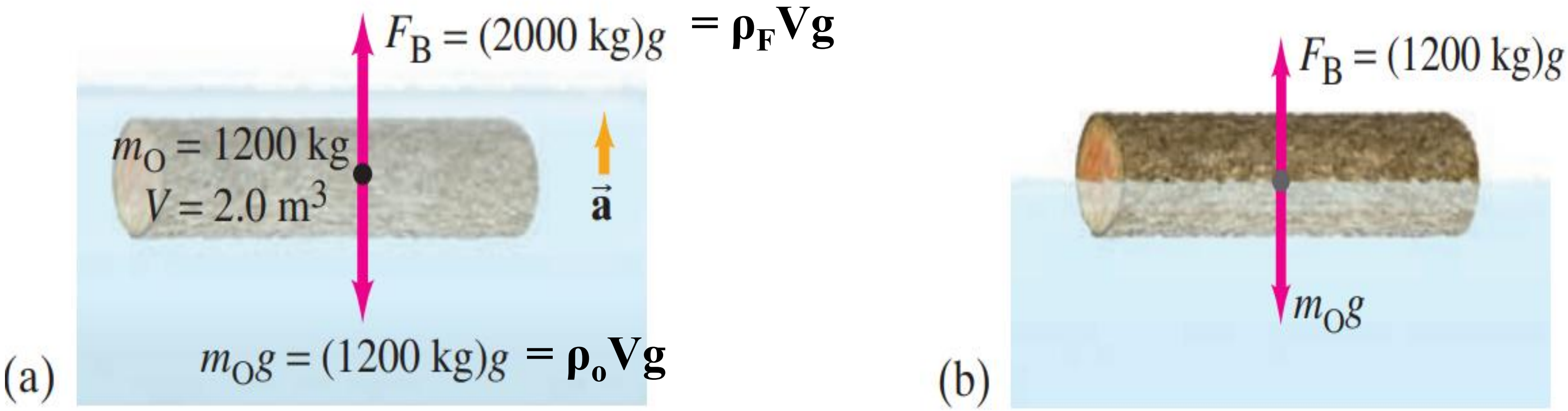


$\Rightarrow f = (V_{displ}/V_o) = (\rho_O/\rho_F)$ **(1)**

f $\equiv$ Fraction of volume of floating object which is submerged.

Note: If fluid is water, right side of **(1)** is specific gravity of object!

- **Example:** Floating log



(a) Fully submerged: $\mathbf{F_B > W}$

$\Rightarrow \sum \mathbf{F_y = F_B - W = ma}$ (It moves up!)

(b) Floating: $\mathbf{F_B = W}$ or $\boldsymbol{\rho_F V g = \rho_O V g}$

$\Rightarrow \sum \mathbf{F_y = F_B - W = 0}$ (Equilibrium: It floats!)

Problem 31:

The specific gravity of ice is 0.917, whereas that of seawater is 1.025. What percent of an iceberg is above the surface of the water?

Prob. 31: Floating Iceberg!

$(SG)_{ice} = 0.917 \equiv (\rho_{ice}/\rho_{water})$, $(SG)_{sw} = 1.025 \equiv (\rho_{sw}/\rho_{water})$

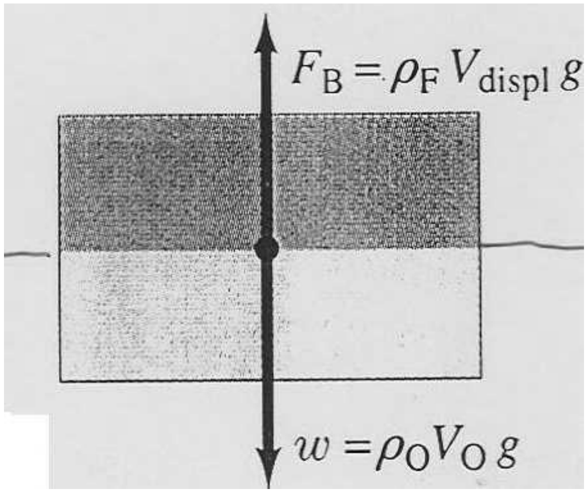
What fraction f_a of iceberg is ***ABOVE*** water's surface? Iceberg volume $\equiv V_O$

Volume submerged $\equiv V_{displ}$

Volume visible $\equiv V = V_O - V_{displ}$

Archimedes: $F_B = \rho_{sw} V_{displ} g$

$m_{ice} g = \rho_{ice} V_O g$



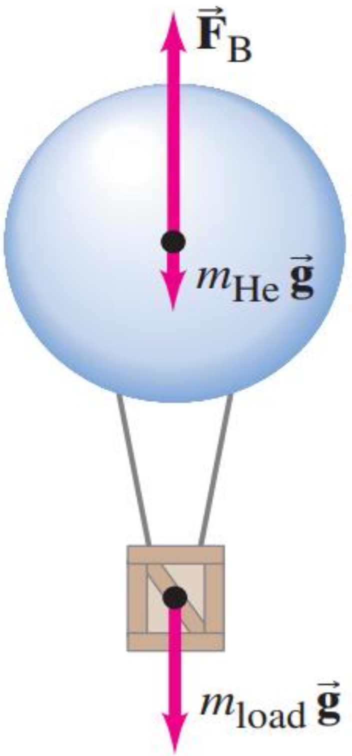
$\sum F_y = 0 = F_B - m_{ice} g \Rightarrow \quad \rho_{sw} V_{displ} = \rho_{ice} V_O$

$(V_{displ}/V_O) = (\rho_{ice}/\rho_{sw}) = [(SG)_{ice}/(SG)_{sw}] = 0.917/1.025 = 0.89$

$$f_a = (V/V_O) = 1 - (V_{displ}/V_O) = 0.11\ (11\%!)$$

Example 10-10:

Helium balloon. What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?



SOLUTION The buoyant force must have a minimum value of

$$F_B = (m_{He} + 180\,\text{kg})g.$$

This equation can be written in terms of density using Archimedes' principle:

$$\rho_{air} V g = (\rho_{He} V + 180\,\text{kg})g.$$

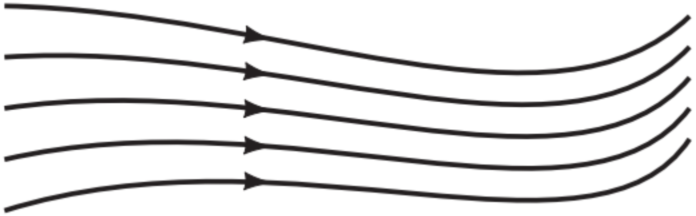
Solving now for V, we find

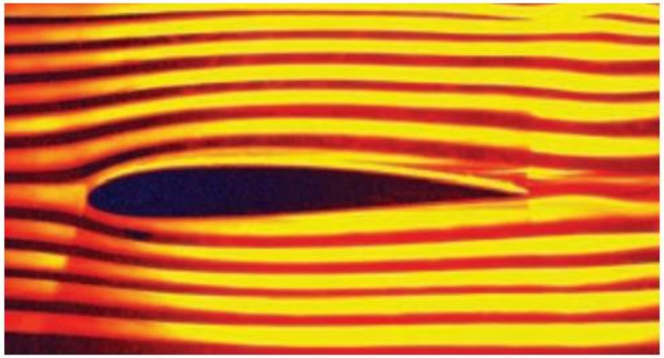
$$V = \frac{180\,\text{kg}}{\rho_{air} - \rho_{He}} = \frac{180\,\text{kg}}{(1.29\,\text{kg/m}^3 - 0.179\,\text{kg/m}^3)} = 160\,\text{m}^3.$$

Sect. 10-8: Fluids in Motion (Hydrodynamics)

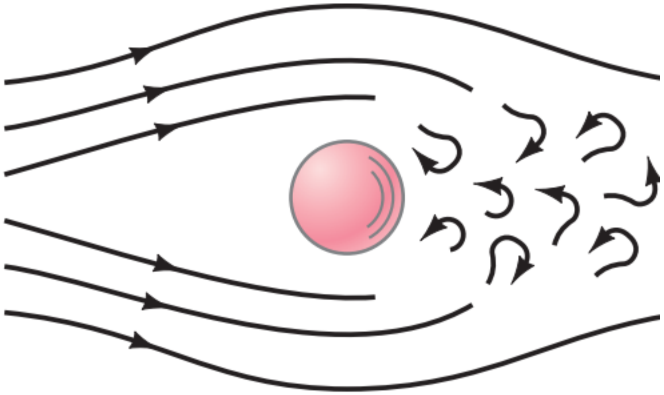
- Two types of fluid flow:

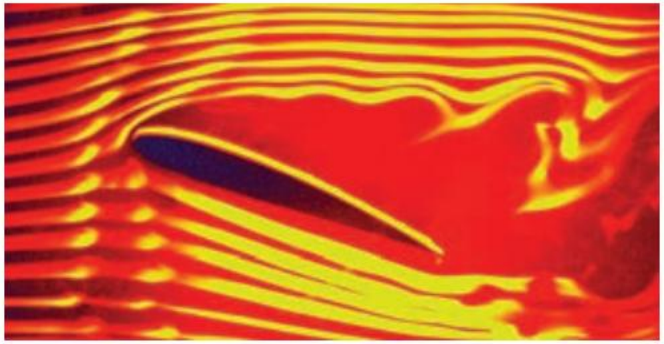
1. Laminar or Streamline: (We'll assume!)





2. Turbulent: (We'll not discuss!)





Streamline Motion

How the speed of the fluid changes when the diameter of the tube changes.

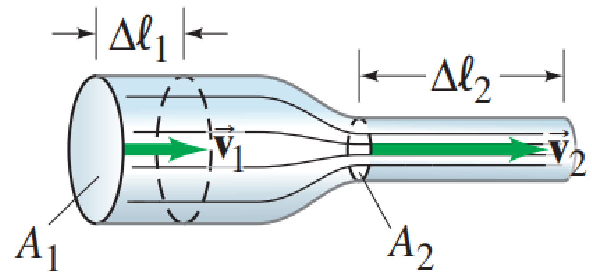


FIGURE 10–19 Fluid flow through a pipe of varying diameter.

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t}$$

The velocity of fluid (density ρ_1) passing point 1 is $v_1 = \Delta\ell_1/\Delta t$

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta\ell_1}{\Delta t} = \rho_1 A_1 v_1$$

Since no fluid flows in or out the sides of the tube, the flow rates through and must be equal. Thus

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This is called the equation of continuity

- Mass flow rate (mass of fluid passing a point per second): $\boldsymbol{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$

$\equiv$ **Equation of Continuity**

***PHYSICS*: Conservation of Mass!!**

- Assume incompressible fluid $(\boldsymbol{\rho_1 = \rho_2 = \rho})$

Then $\Rightarrow$ $\boldsymbol{A_1 v_1 = A_2 v_2}$

Or: $\boldsymbol{Av}$ = **constant**

  - Where cross sectional area **A** is large, velocity **v** is small, where **A** is small, **v** is large.

- Volume flow rate: $\boldsymbol{(\Delta V/\Delta t) = A(\Delta \ell/\Delta t) = Av}$

- ***PHYSICS*:** **Conservation of Mass!!**

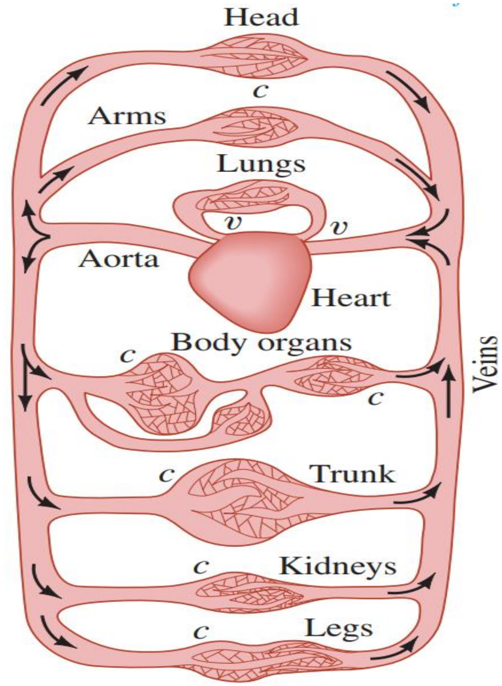
  $\mathbf{A_1 v_1 = A_2 v_2}$ Or $\mathbf{Av = constant}$

- ***Small pipe cross section*** $\Rightarrow$ ***larger*** **v**

- ***Large pipe cross section*** $\Rightarrow$ ***smaller*** **v**

Example 10-12:

EXAMPLE 10–12 ESTIMATE **Blood flow.** In humans, blood flows from the heart into the aorta, from which it passes into the major arteries, Fig. 10–20. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.



SOLUTION Let A_1 be the area of the aorta and A_2 be the area of *all* the capillaries through which blood flows. Then $A_2 = N\pi r_{cap}^2$, where $r_{cap} \approx 4 \times 10^{-4}$ cm is the estimated average radius of one capillary. From the equation of continuity (Eq. 10–4b), we have

$$v_2 A_2 = v_1 A_1$$

$$v_2 N \pi r_{cap}^2 = v_1 \pi r_{aorta}^2$$

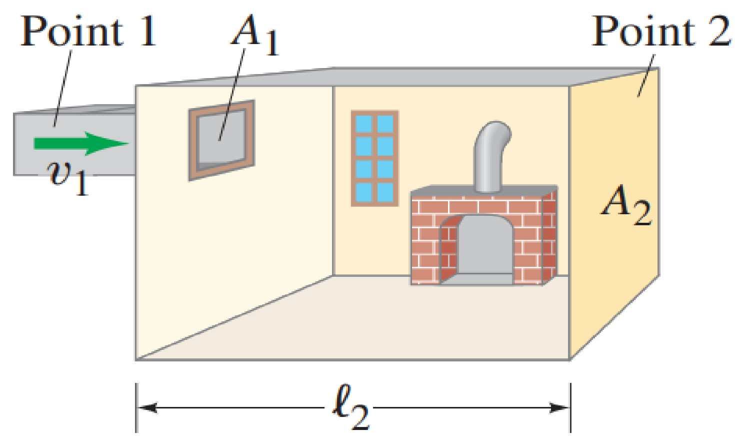
so

$$N = \frac{v_1}{v_2}\frac{r_{aorta}^2}{r_{cap}^2} = \left(\frac{0.40\ \text{m/s}}{5 \times 10^{-4}\ \text{m/s}}\right)\left(\frac{1.2 \times 10^{-2}\ \text{m}}{4 \times 10^{-6}\ \text{m}}\right)^2 \approx 7 \times 10^9,$$

or on the order of 10 billion capillaries.

Example 10-13:

Heating duct to a room. What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3? Assume the air's density remains constant.



Solution:

$$v_2 = \ell_2/t \text{ so } A_2 v_2 = A_2 \ell_2/t = V_2/t$$

V_2 is the volume of the room

$$A_1 v_1 = A_2 v_2 = V_2/t$$

$$A_1 = \frac{V_2}{v_1 t} = \frac{300\ \text{m}^3}{(3.0\ \text{m/s})(900\ \text{s})} = 0.11\ \text{m}^2$$

Section 10-9: Bernoulli's Equation

- **Bernoulli's Principle** (qualitative):

  "Where the fluid velocity is high, the pressure is low, and where the velocity is low, the pressure is high."

  - Higher pressure slows fluid down. Lower pressure speeds it up!

- **Bernoulli's Equation** (quantitative).
  - We will now derive it.
  - ***NOT*** a new law. Simply conservation of KE + PE (or the Work-Energy Principle) rewritten in fluid language!

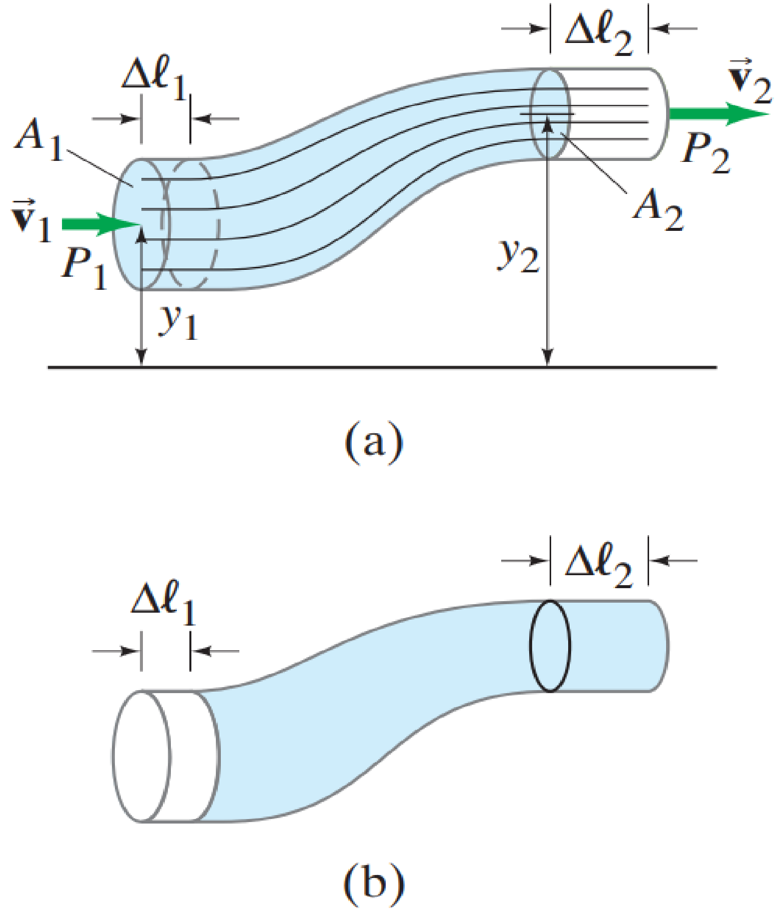


FIGURE 10–22 Fluid flow: for derivation of Bernoulli's equation.

$$P_1 + (½)\rho(v_1)^2 + \rho g y_1 = P_2 + (½)\rho(v_2)^2 + \rho g y_2$$

≡ ***Bernoulli's Equation***

- Another form:

$$P + (½)\rho(v_1)^2 + \rho g y_1 = \text{constant}$$

- Not a new law, just work & energy of system in fluid language. (***Note:*** $P \neq \rho\, g(y_2 - y_1)$ since fluid is ***NOT*** at rest!)

Work Done by Pressure = ΔKE + ΔPE

Sect. 10-10: Applications of Bernoulli's Eqtn

$$P_1 + (½)\rho(v_1)^2 + \rho g y_1 = P_2 + (½)\rho(v_2)^2 + \rho g y_2$$

≡ ***Bernoulli's Equation***

Or:

$$P + (½)\rho(v_1)^2 + \rho g y_1 = \text{constant}$$

NOTE! The fluid is ***NOT*** at rest, so **$\Delta P \neq \rho g h$!**

- Example 10-13

Example 10-14:

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

SOLUTION We take v_2 in the equation of continuity, Eq. 10–4, as the flow speed on the second floor, and v_1 as the flow speed in the basement. Noting that the areas are proportional to the radii squared $(A = \pi r^2)$, we obtain

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 \pi r_1^2}{\pi r_2^2} = (0.50\,\mathrm{m/s})\frac{(0.020\,\mathrm{m})^2}{(0.013\,\mathrm{m})^2} = 1.2\,\mathrm{m/s}.$$

To find the pressure on the second floor, we use Bernoulli's equation (Eq. 10–5):

$$\begin{aligned} P_2 &= P_1 + \rho g(y_1 - y_2) + \tfrac{1}{2}\rho(v_1^2 - v_2^2) \\ &= (3.0 \times 10^5\,\mathrm{N/m^2}) + (1.0 \times 10^3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(-5.0\,\mathrm{m}) \\ &\quad + \tfrac{1}{2}(1.0 \times 10^3\,\mathrm{kg/m^3})[(0.50\,\mathrm{m/s})^2 - (1.2\,\mathrm{m/s})^2] \\ &= (3.0 \times 10^5\,\mathrm{N/m^2}) - (4.9 \times 10^4\,\mathrm{N/m^2}) - (6.0 \times 10^2\,\mathrm{N/m^2}) \\ &= 2.5 \times 10^5\,\mathrm{N/m^2} = 2.5\,\mathrm{atm}. \end{aligned}$$

Application #1: Water Storage Tank

$$P_1 + (½)\rho(v_1)^2 + \rho g y_1 = P_2 + (½)\rho(v_2)^2 + \rho g y_2 \quad (1)$$

Fluid flowing out of spigot at bottom. Point 1 → spigot

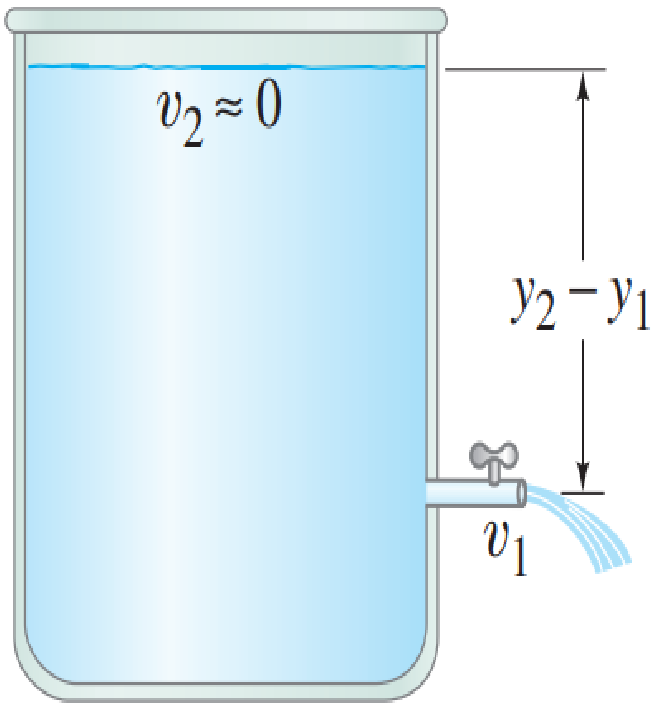
Point 2 → top of fluid

$$v_2 \approx 0 \qquad (v_2 << v_1)$$

$$P_2 \approx P_1$$

(1) becomes:

$$(½)\rho(v_1)^2 + \rho g y_1 = \rho g y_2$$



Or, speed coming out of spigot: $v_1 = [2g(y_2 - y_1)]^{½}$

"Torricelli's Theorem"

Application #2: Flow on the level

$$P_1 + (½)\rho(v_1)^2 + \rho g y_1 = P_2 + (½)\rho(v_2)^2 + \rho g y_2 \quad (1)$$

- Flow on the level $\Rightarrow y_1 = y_2 \Rightarrow$ **(1)** becomes:

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \quad (2)$$

(2) Explains many fluid phenomena & is a quantitative statement of ***Bernoulli's Principle:***

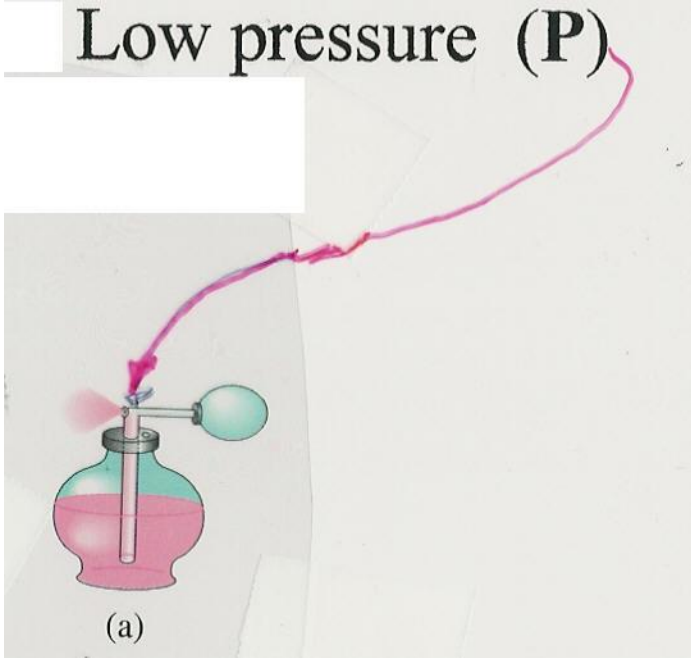
"Where the fluid velocity is high, the pressure is low, and where the velocity is low, the pressure is high."

Application #2 a) Perfume Atomizer

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \quad (2)$$

"Where v is high, P is low, where v is low, P is high."

- High speed air (v) ⇒



(a)

⇒ Perfume is "sucked" up!

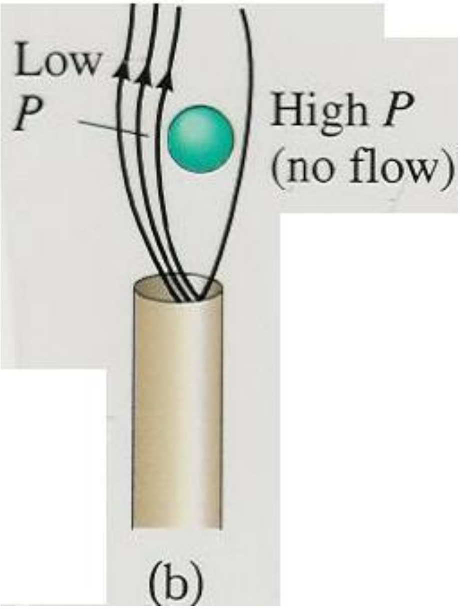
Application #2 b) Ball on a jet of air

(Demonstration!)

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \qquad (2)$$

"Where v is high, P is low, where v is low, P is high."

- High pressure (**P**) outside air jet ⇒ Low speed (**v ≈ 0**). Low pressure (**P**) inside air jet ⇒ High speed (**v**)

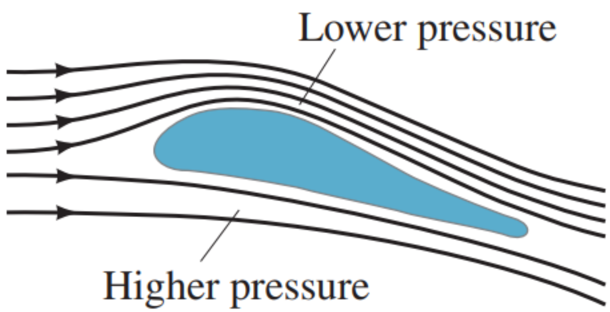


Application #2 c) Lift on airplane wing

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \qquad (2)$$

"Where v is high, P is low, where v is low, P is high."

FIGURE 10–25 Lift on an airplane wing. We are in the reference frame of the wing, seeing the air flow by.



$P_{TOP} < P_{BOT} \Rightarrow$ ***LIFT!***

$A_1 \equiv$ Area of wing top, $A_2 \equiv$ Area of wing bottom

$$F_{TOP} = P_{TOP} A_1 \quad F_{BOT} = P_{BOT} A_2$$

Plane will fly if $\sum F = F_{BOT} - F_{TOP} - Mg > 0$!

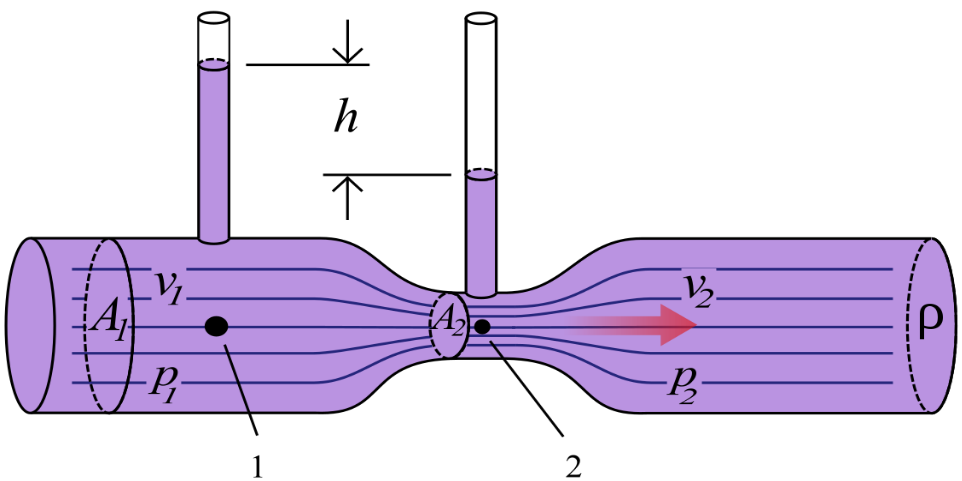
Application #2 e) "Venturi" tubes

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \quad (2)$$

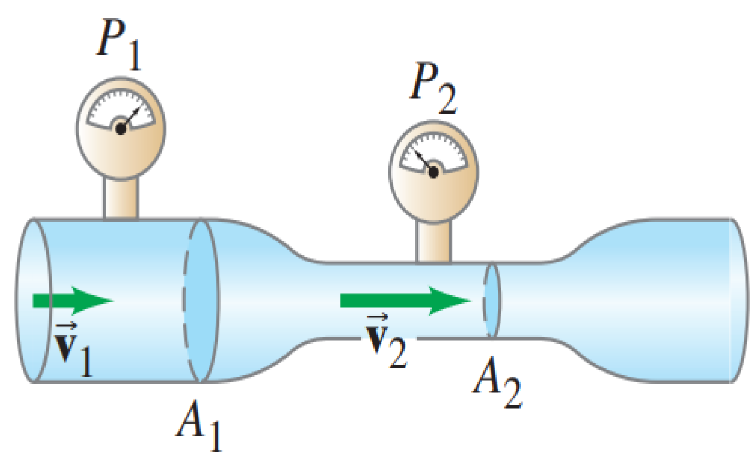
"Where v is high, P is low, where v is low, P is high."

Venturi meter: $A_1 v_1 = A_2 v_2$ (Continuity)

With (2) this $\Rightarrow P_2 < P_1$



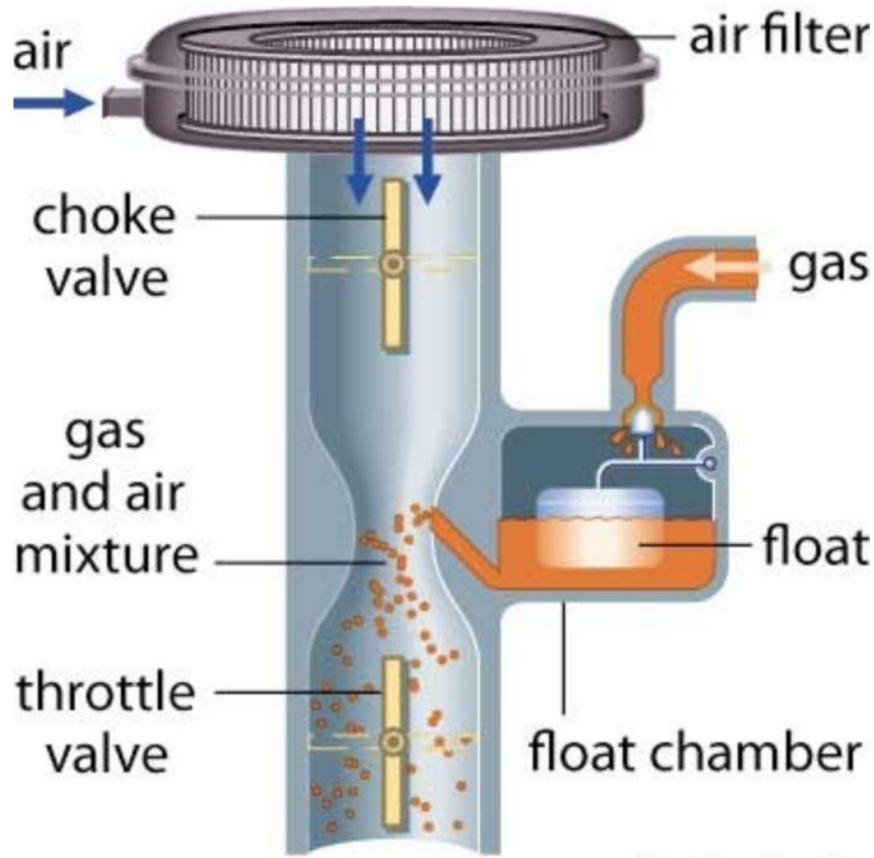
(a)



(b)

How Do Carburettors Work?

For your Knowledge



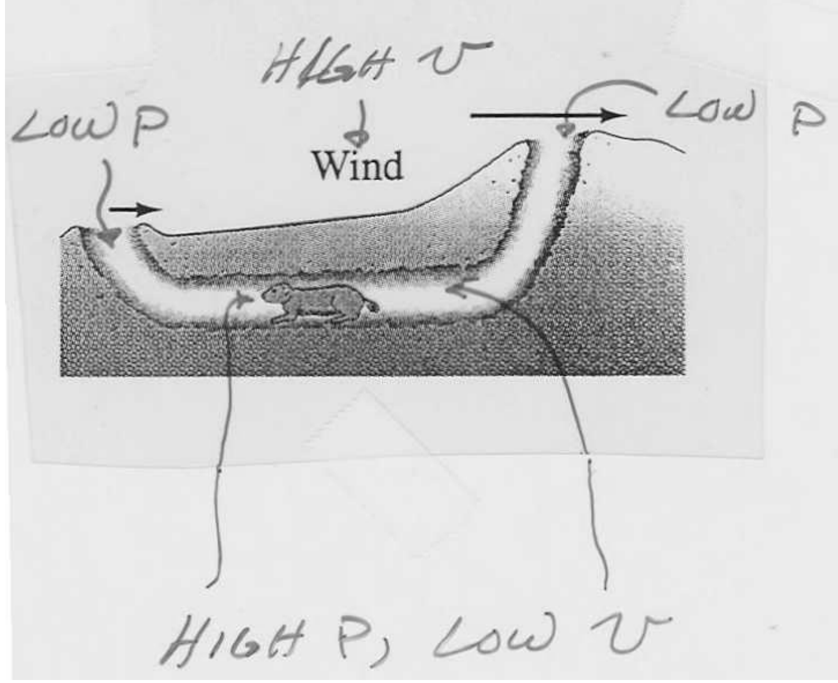
Application #2 f) Ventilation in “Prairie Dog Town” & in chimneys etc.

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \quad (2)$$

“Where v is high, P is low, where v is low, P is high.”

⇒ Air is forced to circulate!

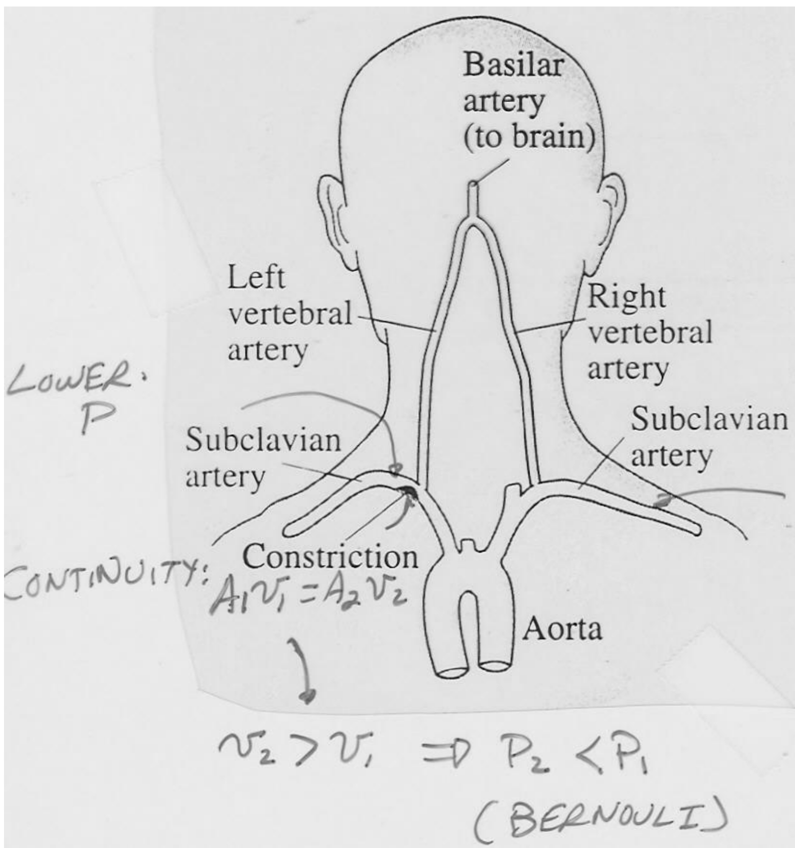
FIGURE 10–27 Bernoulli’s principle is responsible for air flow in underground burrows.



Application #2 g) Blood flow in the body

$$P_1 + (½)\rho(v_1)^2 = P_2 + (½)\rho(v_2)^2 \qquad (2)$$

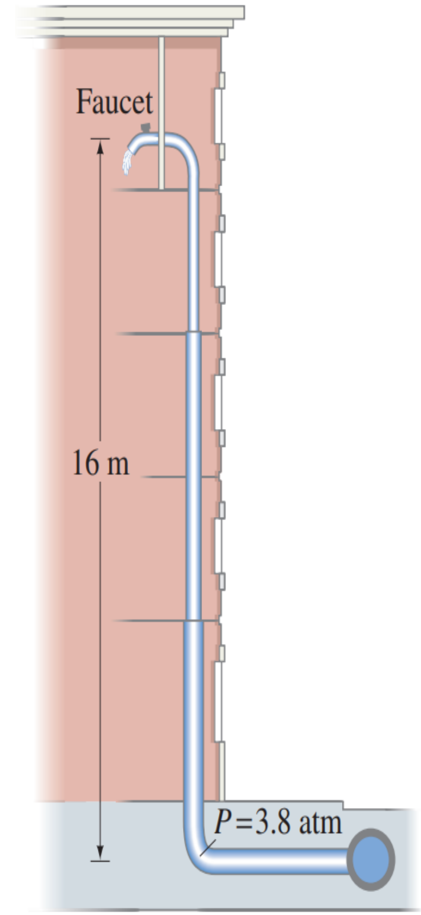
"Where* v *is high,* P *is low, where* v *is low,* P *is high."



$\Rightarrow$ Blood flow is from right to left instead of up (to the brain)

Problem 48: Pumping water up

Water at a gauge pressure of 3.8 atm at street level flows into an office building at a speed of 0.78 m/s through a pipe 5.0 cm in diameter. The pipe tapers down to 2.8 cm in diameter by the top floor, 16 m above (Fig. 10–53), where the faucet has been left open. Calculate the flow velocity and the gauge pressure in the pipe on the top floor. Assume no branch pipes and ignore viscosity



10-12 Flow in Tubes; Poiseuille's Equation, Blood Flow

The rate of flow in a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube.

The volume flow rate is proportional to the pressure difference, inversely proportional to the length of the tube, and proportional to the fourth power of the radius of the tube.

10-12 Flow in Tubes; Poiseuille's Equation, Blood Flow

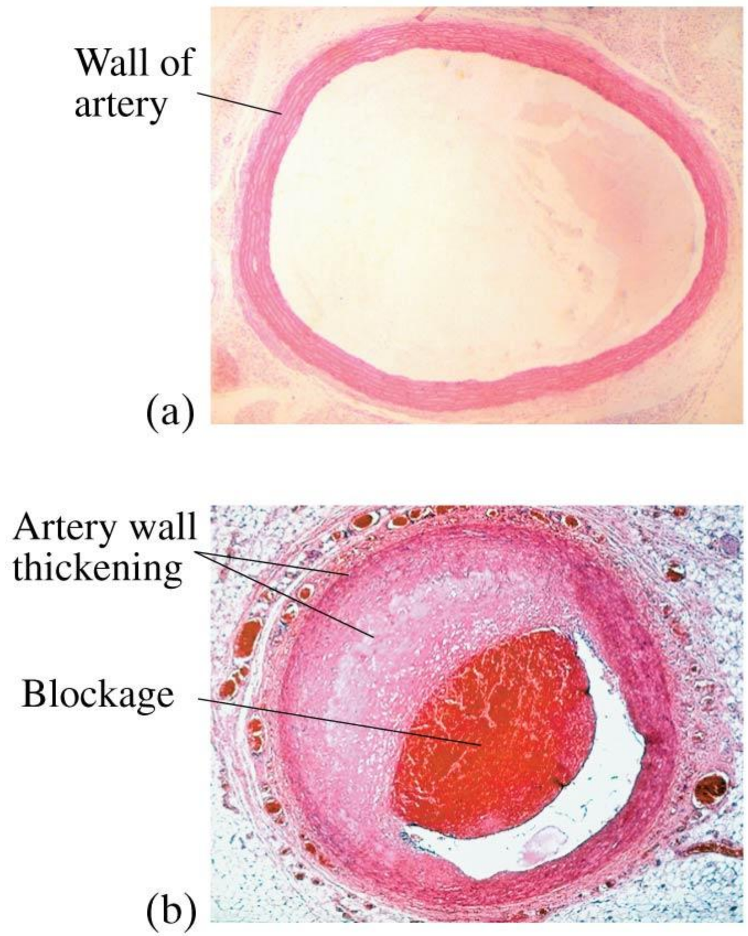
$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta \ell},$$

where R is the inside radius of the tube, ℓ is the tube length, $P_1 - P_2$ is the pressure difference between the ends, η is the coefficient of viscosity, and Q is the volume rate of flow (volume of fluid flowing past a given point per unit time which in SI has units of). Equation 10–9 applies only to laminar (streamline) flow.

10-12 Flow in Tubes; Poiseuille's Equation, Blood Flow

This has consequences for blood flow—if the radius of the artery is half what it should be, the pressure has to increase by a factor of 16 to keep the same flow.

Usually the heart cannot work that hard, but blood pressure goes up as it tries.



Problem 5:

A bottle has a mass of 35.00 g when empty and 98.44 g when filled with water. When filled with another fluid, the mass is 89.22 g. What is the specific gravity of this other fluid?

Problem 10:

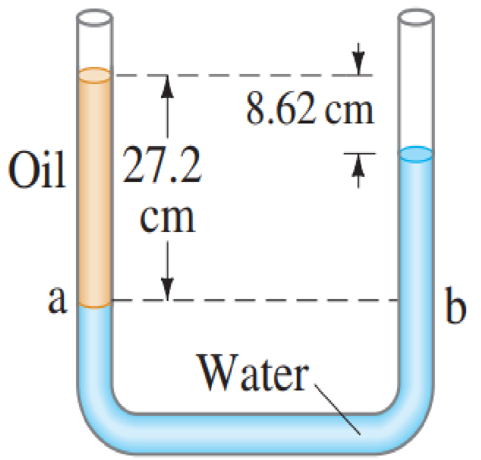
What is the difference in blood pressure (mm-Hg) between the top of the head and bottom of the feet of a 1.75-m-tall person standing vertically?

Problem 11:

(a) Calculate the total force of the atmosphere acting on the top of a table that measures1.7m x 2.6m (b) What is the total force acting upward on the underside of the table?

Problem 18:

Water and then oil (which don't mix) are poured into a tube, open at both ends. They come to equilibrium as shown in Fig. 10–50. What is the density of the oil? [Hint: Pressures at points a and b are equal. Why?]



Problem 20:

Determine the minimum gauge pressure needed in the water pipe leading into a building if water is to come out of a faucet on the fourteenth floor, 44 m above that pipe.

Problem 27:

What is the likely identity of a metal (see Table 10–1) if a sample has a mass of 63.5 g when measured in air and an apparent mass of 55.4 g when submerged in water?

Problem 39:

How fast does water flow from a hole at the bottom of a very wide, 4.7-m-deep storage tank filled with water? Ignore viscosity.

Problem 54:

(I) A gardener feels it is taking too long to water a garden with a $\frac{3}{8}$-in.-diameter hose. By what factor will the time be cut using a $\frac{5}{8}$-in.-diameter hose instead? Assume nothing else is changed.

Problem 60:

*60. (III) A patient is to be given a blood transfusion. The blood is to flow through a tube from a raised bottle to a needle inserted in the vein (Fig. 10–54). The inside diameter of the 25-mm-long needle is 0.80 mm, and the required flow rate is 2.0 cm^3 of blood per minute. How high h should the bottle be placed above the needle? Obtain ρ and η from the Tables. Assume the blood pressure is 78 torr above atmospheric pressure.

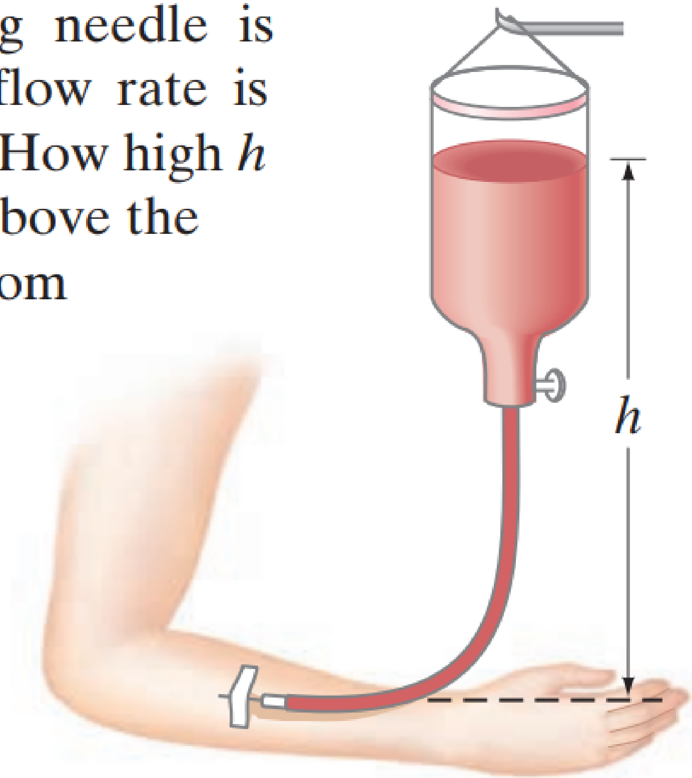


FIGURE 10–54
Problem 60.

Problem 60:

Solution:

The fluid pressure must be 78 torr higher than air pressure as it exits the needle so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 78 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 10–3c to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta_{\text{blood}}\ell} \rightarrow P_2 = P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4} = \rho_{\text{blood}} g \Delta h \rightarrow \Delta h = \frac{1}{\rho_{\text{blood}} g}\left(P_1 + \frac{8\eta_{\text{blood}}\ell Q}{\pi R^4}\right)$$

$$\Delta h = \frac{1}{\left(1050\ \frac{\text{kg}}{\text{m}^3}\right)(9.80\ \text{m/s}^2)}\left(\begin{array}{l}(78\ \text{mm-Hg})\left(\dfrac{133\ \text{N/m}^2}{1\ \text{mm-Hg}}\right) + \\ \dfrac{8(4\times10^{-3}\ \text{Pa}\cdot\text{s})(0.025\ \text{m})\left(\dfrac{2.0\times10^{-6}\ \text{m}^3}{60\ \text{s}}\right)}{\pi(0.4\times10^{-3}\ \text{m})^4}\end{array}\right) = 1.04\ \text{m} \approx \boxed{1.0\ \text{m}}$$